

Counting Handshakes

In this lesson during a summer bridge program, students explored a problem involving systematic counting. This problem was designed to allow for many different approaches, representations, and descriptions of emerging patterns and connections across different examples. The activities were designed to allow for student talk.

As you read this vignette, please keep in mind the following two questions:

- What connections did students make as they tried to solve the problem?
- What could a teacher do to highlight connections between ideas and approaches?

In considering what a teacher might do, it may be helpful to consider the five “steps” for orchestrating productive classroom discussions identified by Smith et al. (2008):

- **Anticipate** the kind of approaches and responses that students might take.
- **Monitor** what students come up with as they engage with the problem.
- **Select** key approaches to bring to the attention of the class during sharing.
- **Sequence** the approaches so there is a progression in terms of generalizability.
- **Connect** during whole-class discussion the approaches, focused on similarities and differences.

Initial Modeling

The teacher displayed the slide for the Handshake problem (Figure 1), and read the text out loud to students.

Patterns

Counting Handshakes

GOAL: Model and solve problems resulting in a sequence of answers

- **Independently Read:** *Starting with two people, how many handshakes can a group of people make so that everyone shakes hands with everyone else?*
- **Independently Draw:** How you would determine the number of handshakes that a group of 3 people and a group of 4 people would make so that everyone shakes hands with everyone else.
- **Partner Discussion:** Discuss how you determined the number of handshakes that a group of 3 people and a group of 4 people would make. Compare drawings.
- **Watch:** Five students act out shaking hands. Before they begin, discuss with your partner how you will count handshakes systematically.
- **Discuss:** Discuss the number of handshakes for 5 people, 10 people, and a rule for the number of people and the number of handshakes.

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26

Figure 1. The Teacher Displayed a Slide with the Handshakes Problem

Students quietly jotted down ideas on their papers independently. At one point, one student softly said:

I go three for three people, if that person, if that hand shakes with that person, with that person, that's another one, and then that's it.

Teacher asks two students to share with their partners:

Eduardo explained, "So I just put three things that indicated the people, and drew arrows to show who they shook hands with."

Sara said, "For me, I draw stick figures representing the people, shaking the vibration of the handshake." (see Figure 2).

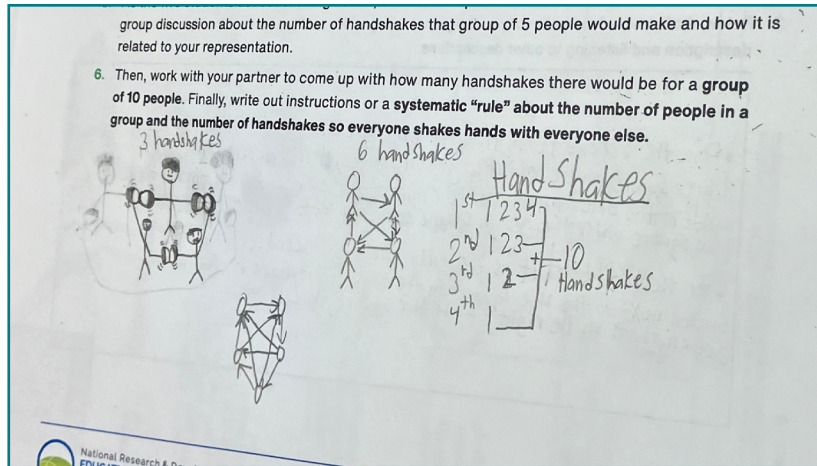


Figure 2. Sara Shared her Representation of Vibrating Handshakes on the Doc Cam

A further student shared a more linear approach and talked through their tallies, "So the 1 2 3 basically are just the people, and then arrows are who they shook hands with." (See Figure 3).

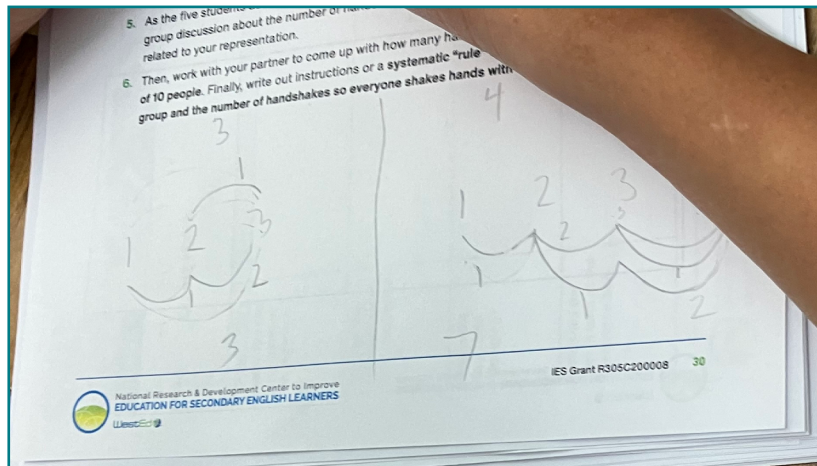


Figure 3. A Student Shared a More Schematic and Numbered Approach

Student: I was trying to figure out how to draw the handshake on the sides.

Teacher: Maybe they stretch around. What did you adjust on your second picture

Student: I drew arrows connecting each person shaking hands.

Publicly Acting Out Handshakes

Five students then stood up and acted out the handshakes. Students called out what they were going to do to record, such as “going to make a chart” and “Tally marks. You can just put in groups I guess.”

The teacher posed a broader question:

Let me pose this question, if you are one of the five people, how would you organize the handshakes? Do you think you can keep track with all of us in the room? How do you think we could systematically, for the five students follow so it’s easier to keep track? Maybe lining up, all five people, all keeping track that way, or maybe it’s a different design

The students demonstrated the handshakes by walking down the line. The first student shook hands down the line and then called the name of the second student who proceeded to shake hands with the remaining student.

A student talked about their diagram:

Well, the first person, the person did four handshakes, the second person went down and did 3 then the third person did 2, then the fourth person only did one handshake, and I added them all together.

Another student described a more incremental approach:

I connected them by doing the arrows, and for the bottom person, the fifth person, I made him do the two on the top, since those two didn’t do those two handshakes, so each person handshake with each other.

A student said, “I skipped one on accident.” The teacher replied, “Can you record that, in words?” The student realized that he had only drawn six arches in the third row from the bottom, rather than the 7 that he would have predicted.

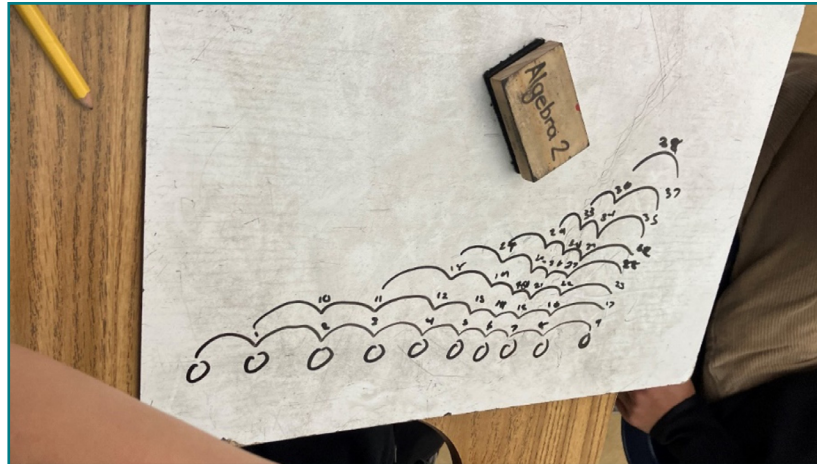


Figure 4. One Student Drew Arches to Represent Each Individual's Handshakes, Numbered Sequentially

Teacher: Kind of lining up, with the tallies, here's another example, first person four check marks.

number of handshakes so everyone shakes hands with everyone else.

	1	2	3	4	5	6	7	8	9	10	
1		✓	✓	✓	✓	✓	✓	✓	✓	✓	9
2	✓		✓	✓	✓	✓	✓	✓	✓	✓	8
3	✓	✓		✓	✓	✓	✓	✓	✓	✓	7
4	✓	✓	✓		✓	✓	✓	✓	✓	✓	6
5	✓	✓	✓	✓		✓	✓	✓	✓	✓	5
6	✓	✓	✓	✓	✓		✓	✓	✓	✓	4
7	✓	✓	✓	✓	✓	✓		✓	✓	✓	3
8	✓	✓	✓	✓	✓	✓	✓		✓	✓	2
9	✓	✓	✓	✓	✓	✓	✓	✓		✓	1
10	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	0

5 ppl each $4 \times = 20$

45 total

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Figure 5. One Student Created a Table to Track and Count Handshakes Systematically

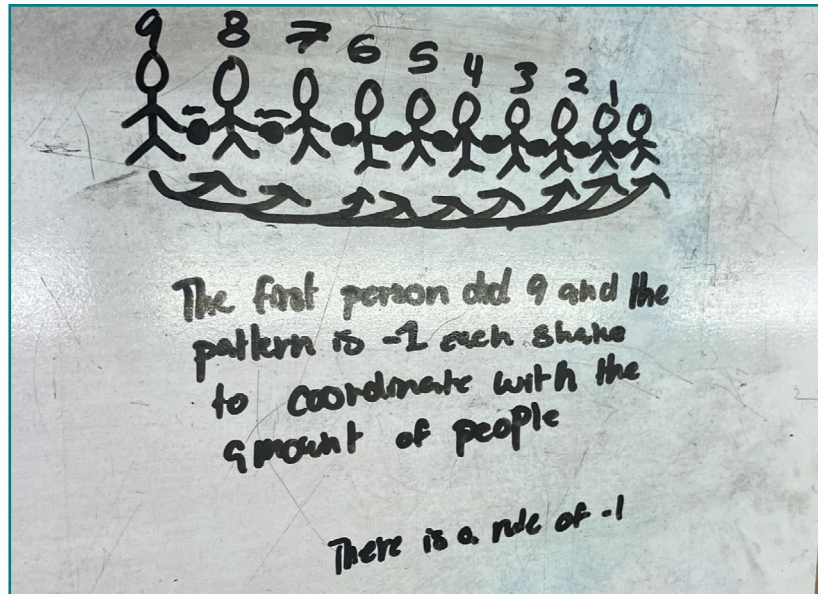


Figure 6. Another Student Wrote Out an Explicit Rule for the Number of Handshakes