National Research & Development Center to Improve
EDUCATION FOR SECONDARY ENGLISH LEARNERS
WestEd

STUDENT MATERIALS

Modeling Networks and Surfaces

Student Name: ______________________________

Class: ______________________________

Module Overview

In this module, you will learn about different ways to represent, analyze, and compare networks. Networks are one way to think about the connections between things. You will apply what you learn about networks to explore how to prove statements mathematically and how to connect different geometric objects.

Mapping Networks → **Counting Across Dimensions** → **Building Up & Breaking Down** → **Proving Platonic Solids**

Notice and Wonder: Card Trick

Goal: Make and connect observations on networks from card tricks.

1. Your teacher will demonstrate a card trick. Observe carefully across the three rounds, and record what you notice and wonder about what makes the trick work.
2. Work independently and record any details and observations related to both noticings and wonderings across the three rounds about how the trick works. Record any connections among your observations.
3. After observing and taking notes on the three rounds, take turns sharing an idea with your partner. After one student offers an idea, another student will share another idea.

Notice and Wonder: Card Trick

	Round 1	Round 2	Round 3
Noticings • details • observations			
Wonderings • questions • information needed			

The Utilities Problem: Round 1

Goal: Solve a problem drawing different connections between objects.

You and your classmates will work together to solve a problem that asks you to complete a task to meet certain conditions. As you try different approaches, ask yourself the following questions:

- What assumptions are you making?
- What other conditions could you try or change to make this possible?

The Utilities Problem: Round 1

The Utilities Problem

There are three houses and three utilities that deliver vital services. These utilities are gas, water, and electricity.

Can you connect all three houses to all three utilities without any of the lines crossing each other?

The Utilities Problem: Round 2

Goal: Solve a problem drawing different connections between objects.

Consider what would happen if you could try to solve the utilities problem on the surface of the earth. Of course, it's not easy to draw the surface of the earth on a sheet of paper, so make a large rectangle and think about how you can "go around" the world with your lines of connection. Show your work below.

The Utilities Problem: Round 2

The Utilities Problem

There are three houses and three utilities that deliver vital services. These utilities are gas, water, and electricity.

Can you connect all three houses to all three utilities without any of the lines crossing each other?

The Utilities Problem: Round 3

Goal: Solve a problem drawing different connections between objects.

Consider other ways that the connections could go around each other on the flat map. Explain how the connections "go around" in different ways. How does this make your solution possible?

The Utilities Problem: Round 3

The Utilities Problem

There are three houses and three utilities that deliver vital services. These utilities are gas, water, and electricity.

Can you connect all three houses to all three utilities without any of the lines crossing each other?

Novel Ideas Only

When I hear the word “network,” I think of ...

Introducing Networks

Goal: Understand networks, how to represent them, and how to use them to map relationships.

Part 1: Reading with Clarifying Bookmark III

The text on the next page provides new information about networks and how they present structure. Use the *Clarifying Bookmark III*, as needed, to read and answer the questions with your partner.

Part 2: Drawing Networks

Draw some of your social networks. Look at the picture and ask:

- Who is at the center? How can you show this?
- What is the diameter? How can you show this?
- What would the “radius” be? How can you show this?

Clarifying Bookmark III

What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Introducing Networks

Connecting the World with Wires

What are networks and how can we represent them?

In everyday life, there are many connections between different people, places, and things. It is important to have a way to show the connections between these different objects. In mathematics, the study of these networks of connections is called "graph theory." The networks are represented as graphs that show pictures of the connection between objects.

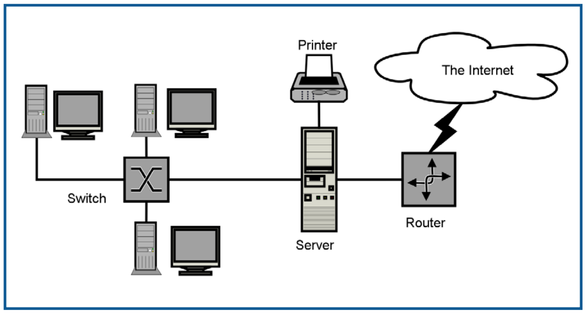


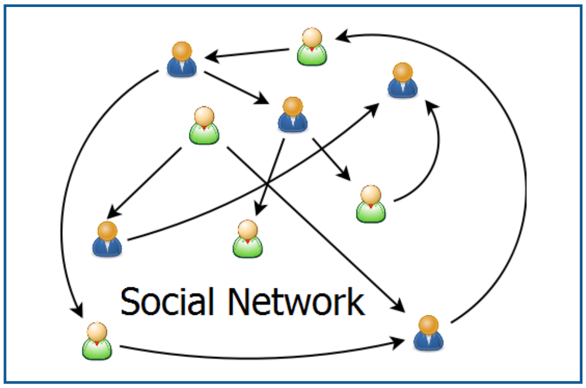
Image attribution: SilverStar at English Wikipedia

One common example of a network is the internet, which is built on large servers. Each image attribution: SilverStar at English Wikipedia of these servers is a large computer that stores information such as websites and videos These servers connect to each other and individual users. All the elements of the server use addresses to connect and exchange data. A "map" of the internet thus could include a map of all of the servers, users, and other points of connection. The users, servers, and connectors are the points, while the lines represent the different connections.

Seeing Social Networks

How can networks map social relationships?

Just as the physical and electronic structure of the internet connects different computers, people can also use the internet to connect to one another. It is possible to map the "social network" of individuals based on their friendships, whether these exist on the internet or in real life. Different social networks make different assumptions—on some networks you "follow" others. In other networks, the relationship of friendship is more mutual.



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There have also been efforts to do this in terms of movie stars and modern social media such as TikTok. Two actors or actresses count as connected if they were in the same movie. From this relationship, it is possible to construct a network that connects actors with each other. There is also a game known as "six degrees of separation," where someone would try to use no more than six movies to connect one given actor to another. In this case, the given actor would be the center of a network. The "diameter" of a social network is then the greatest distance between two individuals as measured by connections, for example from one actor to another, just like the diameter of a circle is the greatest distance between two points. With TikTok, you could try and find the most popular content creator based on who has done duets with each other.

Being in the same duet would count as a connection, so finding the "center" would be finding the person who is connected to everyone else through the smallest number of possible connections.

Daily Writing Prompt: Mapping Networks

We started today making and drawing connections to solve problems, and then we related this work to networks and mapping. What additional connections do you now see in the card trick problem that you did not see before? What questions do you have about how to create networks?

Sort and Label: Networks

Goal: Describe, sort, and label networks in the everyday world.

1. **Describe.** The first student will take one card and describe it to the group without showing them the card. After describing their card, the student should place it on the table for all group members to see.
2. **Sort.** The next student will take another card, describe it, and share if it is similar to or different from the other card and how. As students describe their cards, they will suggest whether the new card belongs or not to a particular group. You may also start new groups of cards as you work.

   You may find the following language helpful as you work:

   - *These cards should be in the same group because ...*
   - *I think these cards belong to different groups because ...*
   - *I think this card needs a new group because ...*

3. **Label.** After all the cards are placed on the table, discuss whether the groups need to be changed. Then describe each group with a short label and write the label on a sticky note.

Defining and Identifying Dimensions

Goal: Understand and identify how geometrical ideas are used in networks.

How Are Geometrical Ideas Used in Networks?

Just as in geometry, where there are points, lines, and polygons, it is possible in the study of networks to define and count different dimensions. The following table shows the different levels.

Geometry	Networks	Example: Social Network	Example: Polyhedron
Point	Vertex	An individual person	The six corners of a cube
Line/Segment	Edge	A friendship between two people	The twelve edges of a cube
Face/Polygon	Face	A closed loop of friends	The six surfaces of a cube

One rule for deciding whether something is a face is to see if you can “color” in the space completely. In general, we only look at connected networks, as you could have as many disconnected vertices as you like, but that would not really make a network.

Count and Record

Review the networks below and identify the number of vertices, edges, and faces.

Network	Number of Vertices	Number of Edges	Number of Faces

How Do Regular Shapes Look in Three Dimensions?

When you studied geometry, you learned about regular polygons. These are shapes that have all congruent sides and all congruent angles, such as an equilateral triangle or a square. When we extend into the third dimension and then allow for shapes to have height or depth as well, we can also look for such shapes, called *polyhedra*, from the ancient Greek term for "many faces." For example, the most familiar regular polyhedron (where all the faces are regular polygons) is probably the cube.

In a cube, each of the six faces is a square. Given this information, you can count the total number of corners, edges, and faces. These values are one way of describing the cube, though other polyhedra that are not regular would also have the same values. The ancient Greeks also studied the regular polyhedra and were able to prove there are five—and only five—such solids, which are often called "Platonic" after the philosopher Plato.

Scribble, Pass, and Count

Goal: Explore and analyze scribble graphs.

You will work in a group of four during this activity.

1. **Scribble a squiggle.** Each student will draw a closed, self-intersecting loop. Then, each student will hand their drawing to the person on their left.
2. **Choose and count.** For each graph, the student will choose a level of analysis (vertices, edges, or faces). They will count and report this to the next student and pass it to them.
3. **Choose and count.** Then, the next student will choose a different level of analysis (vertices, edges, or faces), count, and report this to the next student, passing it to them.
4. **Predict and check.** The last student will make a prediction for the last level of analysis (vertices, edges, or faces), without counting. Then, students will check their predictions.

Two Problems

Goal: Solve two problems involving making and drawing different connections.

You and your classmates will work together to solve two problems. Each problem asks you to complete a task to meet certain conditions. As you try different approaches, ask yourself the following questions:

- What assumptions are you making?
- What other conditions could you try or change to make this possible?

Two Problems

Handshakes Among Five People

Previously, we were able to count the handshakes among five people. We were able to come up with a formula for the number of handshakes using a diagram.

Can you draw all 10 handshakes among five people without any of the lines crossing?

Two Problems

Venn Diagram of Four Sets

Venn diagrams show the overlaps or intersections between different sets or characteristics. For example, when looking at cars, you may consider those that are blue (or not blue), and that have four doors (or two doors). Similarly, consider these three characteristics, each yes or no for selecting a movie:

- Longer than two hours?
- Action?
- In English?

For each of these, it is possible to draw circles to show all the possibilities and combinations.

Can you draw four circles to show all the possibilities and combinations across four different questions?

Daily Writing Prompt: Counting Across Dimensions

Today we learned more about how to label and count networks. What patterns, if any, do you notice in the counts of vertices, edges, and faces? What questions do you have about networks and these dimensions?

Step it Up (Building Up)

Goal: Co-create networks to identify how the number of vertices, edges, and faces change and relate.

In this activity, you will work with your partner to build up and break down networks.

1. With your partner, determine who will be Student A and who will be Student B. Student A will begin by drawing a network. Then Student B will propose adding an additional element or elements to make a connected network.
2. Student B will then record the new network in the following row, as well as the number.
3. **Take turns** suggesting changes and drawing the next network. Record the values in the columns of the table as you go.
4. **Discuss the networks with your partner.** When you and your partner have completed all networks, discuss what you learned.

Consider the following questions as you work:

- How are the options of what to add—a vertex, edge, or face—different depending on the current network?
- How are the number of edges changing?

Step it Up (Building Up)

	Network	V	E	F
A				
B				
A				
B				
A				
B				
A				
B				

Step it Up (Breaking Down)

Goal: Analyze networks by deconstructing them in order to relate changes between the numbers of vertices, edges, and faces.

In this activity, you will work with your partner to build up and break down networks.

1. With your partner, take turns quickly drawing a complex network. Make sure the work has at least 10 vertices. Record the numbers of vertices, edges, and faces.
2. Determine who will be Student A and who will be Student B. Student A will begin by proposing parts to delete. Student B will follow that suggestion, draw the new network, and record the numbers of vertices, edges, and faces.
3. Student B will then propose a new deletion to make, which student A will carry out.
4. **Take turns** suggesting deleting and drawing the next, smaller network.
5. **Discuss the networks with your partner.** When you and your partner have completed all networks, discuss what you learned.

Consider the following questions as you work:

- How are the options of what to add—a vertex, element, or face—different depending on the current network?
- How are the number of edges changing?

Step it Up (Breaking Down)

	Network	V	E	F
A				
B				
A				
B				
A				
B				
A				
B				

Two Problems Revisited

Goal: Solve two problems involving making and drawing different connections.

Work with a partner on the two problems on the next two pages.

Two Problems Revisited

Handshakes Among Five People

Previously, we were able to count the handshakes among five people. We were able to come up with a formula for the number of handshakes using a diagram.

Can you draw all 10 handshakes among five people without any of the lines crossing?

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Two Problems Revisited

Venn Diagram of Four Sets

Venn diagrams show the overlaps or intersections between different sets or characteristics. For example, when looking at cars, you may consider those that are blue (or not blue), and that have four doors (or two doors). Similarly, consider these three characteristics, each yes or no for selecting a movie:

- Longer than two hours?
- Action?
- In English?

For each of these, it is possible to draw circles to show all the possibilities and combinations.

Can you draw four circles to show all the possibilities and combinations across four different questions?

Compare and Contrast the Two Problems

Complete the chart below about the assumptions that you are making and the conditions you could try.

	What assumptions are you making?	What other conditions could you try or change to make this possible?
Handshakes Among Five People		
Venn Diagram of Four Sets		

Part 1: Data on the Three Problems

1. **Counting what is.** With your partner, review each of the two problems above. It is often helpful in mathematics to consider a simpler case. Specifically, you can draw the diagrams on a flat sheet of paper for the two simpler problems below. Identify what the values of V, E, and F would be if you could successfully complete those tasks. Complete the table below.

Network	V	E	F
Handshakes Among Four People			
Venn Diagram of Three Sets			

2. **Counting what should be.** Now, extend each of the problems by adding an additional person or an additional set. Since you are trying to see if this is possible, it will not be enough to count what you see in a picture that you have drawn. Rather, you will need to make a prediction about what will be necessary in order to successfully draw that picture.

Network	V	E	F
Handshakes Among Five People			
Venn Diagram of Four Sets			

3. **Discuss:** What does this say about the patterns that you have noticed in your construction and deconstruction tasks?

Part 2: Euler Characteristic

The Euler characteristic is the alternating sum of the values we have been working with for any given network:

$$V - E + F$$

What do you notice about the Euler characteristic of any network drawn on a flat surface? What about the surface of a sphere?

Considering Other Surfaces

So far, we have tried to solve problems on familiar surfaces such as the flat plane (like a sheet of paper) and the sphere (like the surface of the earth). Mathematicians have identified other surfaces of note, such as the torus (the surface of a donut or bagel).

1. With your partner, consider the following. To construct just the surface, rather than the solid inside the donut, we can use a sheet of paper and imagine it wrapping around the surface of a flat round object with a hole. When this paper is wrapped, how do the edges of the paper connect to one another?

2. If you were to draw a network with four vertices and edges that go over the wrapped portions, what would the key values and the Euler characteristic be?

Daily Writing Prompt: Building Up and Breaking Down

Look back at how you were building up and breaking down different networks. How does the step-by-step process of either construction or deletion help you to see why the Euler characteristic does not change? Write a brief explanation of why, illustrating some cases of the different kinds of additions or deletions that you could make.

Tessellations in the Plane

One idea that is related to Platonic solids will be that regular polygons can be used to completely fill in the plane. This concept was one that was investigated long ago, also by the ancient Greeks. To help you see the connections, it may be useful to fill out the following table.

Number of Sides	Measure of Exterior Angles	Measure of Interior Angles	Number of Polygons Meeting a Single Point (if possible)
3			
4			
5			
6			
7			
8			
9			
10			

Platonic Solids

Let's return to the case of the cube and see what else we can count. We've already counted vertices, edges, and faces, but part of what makes a shape regular is how each dimension connects to the next. For example, we can ask, "How many vertices for each edge?" and "How many edges for each vertex?" Similarly, we can ask, "How many edges for each face?" and "How many faces for each edge?" Complete the following table with the information that you think pertains to each question.

Question	Answer or Possible Values	Interpretation or Explanation
How many vertices for each edge?		
How many edges for each vertex?		This value, if it is constant, is known as the "degree" of the graph.
How many faces for each edge?		
How many edges for each face?		This value is related to the kind of polygon that is created.

Try to answer this question for the cube. Then add other rows for the other polyhedra you know.

Polyhedron	V	d	E	n	F

Exploring Duality

Which pairs of Platonic solids seem the most related to one another? Why?

Writing Extension Activity

Goal: Construct arguments for why certain problems are either impossible or why you have found all possible cases of something.

Over the course of this module, you have explored multiple networks and considered two kinds of arguments. The first is why certain problems are impossible to do on either a flat plane or the surface of a sphere. You have explored the following problems:

- Three Utilities
- Five Handshakes
- Four Sets in a Venn Diagram

In addition, you have explored five different Platonic solids and verified that each one exists. You may also choose to construct an argument explaining why no other Platonic solids are possible.

Now is your opportunity to demonstrate your individual understanding by completing an extended piece of writing that explains at least one of these problems in depth.

Writing Extension Activity Rubric

Category	Indicators of High-Quality Work	Strengths	Areas to Improve or Revise
Content	• Explanation of the Euler characteristic and why it does not change for any particular surface. • Introduction to the problem, the assumptions it makes, and the key characteristics in terms of vertices, edges, and faces. • Argument for why something is impossible based upon the characteristics and values that are offered.		
Design	• The use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

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