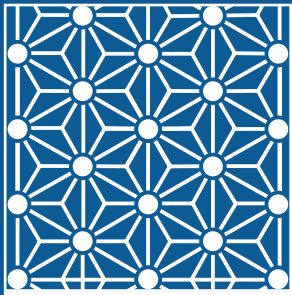




National Research & Development Center to Improve
EDUCATION FOR SECONDARY ENGLISH LEARNERS
WestEd 



STUDENT MATERIALS

Patterns of Growth and Change

Student Name: _____

Class: _____



STUDENT MATERIALS

Patterns of Growth and Change

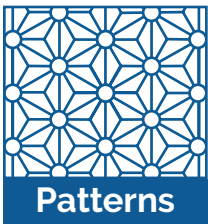
Letter from the Authors

Welcome!

We are delighted to invite you into this course, which is a different way of thinking about and doing mathematics. You have studied mathematics your entire school career, but this course will take a somewhat different approach.

We approach the study of mathematics as the study of structure—the connections and patterns you can use to read and write the world. At the same time, these patterns and structures are interesting to study on their own. Math is both pure and applied.

We want you to embark upon this new journey with a fresh start. Many of your past experiences with math may have been about speed in computations or doing complicated procedures. We instead want you to think about mathematics as a search not just for how but for why.

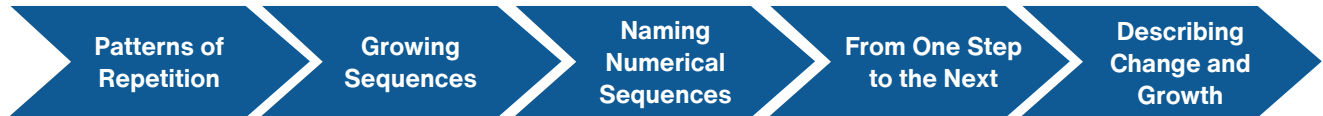


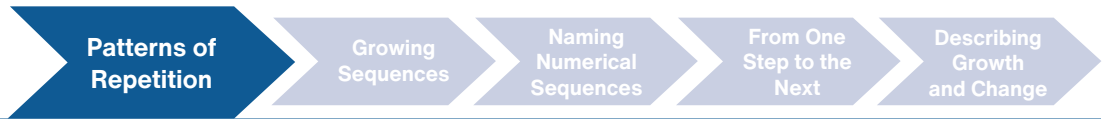
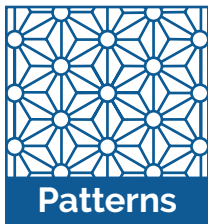
STUDENT MATERIALS

Patterns of Growth and Change

Module Overview

In this module, you will explore many different kinds of patterns. Along the way, you will use mathematical representations, including: real-world situations, geometrical shapes, numerical sequences, and algebraic formulas. You and classmates will begin with a focus on patterns of repetition, and eventually explore patterns that change. As you engage with your classmates, you will grow more sophisticated in linking different parts of mathematical representations while looking for underlying structure and regularity in patterns.



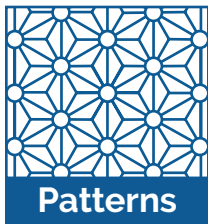


Self-Reflection: Math and Me

The following self-reflection questionnaire serves to help you think about your relationships with mathematics. Read each sentence and rate how much you agree with each sentence, selecting from strongly disagree (SD), disagree (D), agree (A), or strongly agree (SA).

	SD	D	A	SA
I make excellent grades on math tests				
I have always been successful with math				
Even when I study very hard, I do poorly in math				
I got good grades in math on my last report card				
I do well on math assignments				
I do well on even the most difficult math assignments				
Seeing adults do well in math pushes me to do better				
When I see how my math teacher solves a problem, I can picture myself solving the problem in the same way				
Seeing kids do better than me in math pushes me to do better				
When I see how another student solves a math problem, I can see myself solving the problem in the same way				
I imagine myself working through challenging math problems successfully				
I compete with myself in math				
My math teachers have told me that I am good at learning math				

	SD	D	A	SA
People have told me that I have a talent for math				
Adults in my family have told me what a good math student I am				
I have been praised for my ability in math				
Other students have told me that I'm good at learning math				
My classmates like to work with me in math because they think I'm good at it				
Just being in math class makes me feel stressed and nervous				
Doing math work takes all of my energy				
I start to feel stressed out as soon as I begin my math work				
My mind goes blank, and I am unable to think clearly when doing math work				
I get depressed when I think about learning math				
My whole body becomes tense when I have to do math				



Patterns of
Repetition

Growing
Sequences

Naming
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Sequences

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Novel Ideas Only

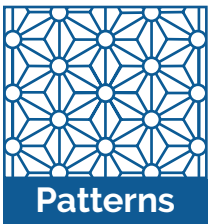
Goal: Brainstorm all the ideas we already have about patterns.

Part 1: Small Group Brainstorming

1. Take turns sharing an idea in response to the prompt, “When I hear the word ‘pattern’, I think of ...”. Each time, someone should read the prompt out loud, then offer an idea.
2. After one student offers an idea, another student will repeat that idea out loud, and all students in your small group will write down the exact same information below. Everyone in the group should have the exact same information on their papers.
3. Continue sharing ideas, repeating them out loud, and writing them down. When time is up, draw a line under the last idea written down in your list.

Part 2: Full Class Sharing-Out

1. Groups will all stand up. One student from each group will read the prompt out loud and then list off their group’s ideas.
2. The other groups must listen carefully as groups share. When your group shares, do not repeat any of the ideas that have already been shared. You can keep track of the ideas from your list that others share by crossing off the lightbulb if another group has already shared out your idea.
3. After a group has shared, they sit down. Group members must still keep listening, however, as they may individually add a few new ideas they really like to their lists.



Patterns of
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Naming
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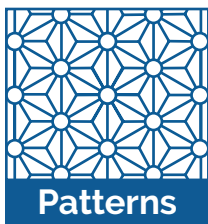
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Novel Ideas Only

When I hear the word "pattern," I think of ...	
	
	
	
	
	
	
	
	



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Sort and Label

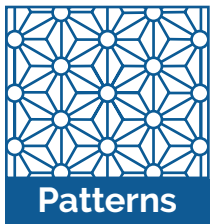
Goal: Describe, sort, and label patterns in the everyday world.

- 1. Describe.** The first student will take one card and describe it to the group without showing them the card. After describing their card, the student should place it on the table for all group members to see.
- 2. Sort.** The next student will take another card, describe it, and share if it is similar to or different from the other card and how. As students describe their cards, they will suggest whether the new card belongs or not to a particular group. You may also start new groups of cards as you work.

You may find the following language helpful as you work:

- *These cards should be in the same group because ...*
- *I think these cards belong to different groups because ...*
- *I think this card needs a new group because ...*

- 3. Label.** After all the cards are placed on the table, discuss whether the groups need to be changed. Then, describe each group with a short label, and write the label on a sticky note. Remember that a good label will be a title that identifies something important about all members of that group.



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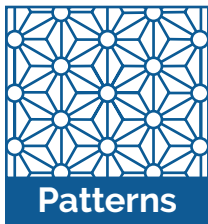


Classmate Bingo Card

Goal: Learn about your classmates and the skills and strengths that they bring to the mathematics classroom.

1. Read all the Bingo squares. Add your name to at least one square that represents a skill or strength that you bring to the mathematics classroom.
2. Ask other classmates for their skills and write their name in the Bingo square with that skill. You can't use the same classmate for more than two squares!





Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

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Classmate Bingo Card

I am good at making and keeping schedules.	I keep my papers from school organized.	I take detailed notes during class.	I can draw well.	I work well with other students.
I ask questions when I don't know the answer.	I am a good listener.	I can speak more than one language.	I always finish projects.	I like art and/or design.
I am a leader during group work.	I enjoy sharing my ideas with others.	FREE	I solve problems in creative and unusual ways.	I can focus on one task without getting distracted.
I take an open-minded approach and enjoy learning new things.	I think about the consequences before I make a decision.	I am courteous to others.	I take turns when in a group or playing a game.	I welcome new people to participate in activities.
I share my ideas with others.	I enjoy reading out loud.	I like to take on a challenge.	I can summarize ideas from a story or reading.	I am persistent when schoolwork is challenging.



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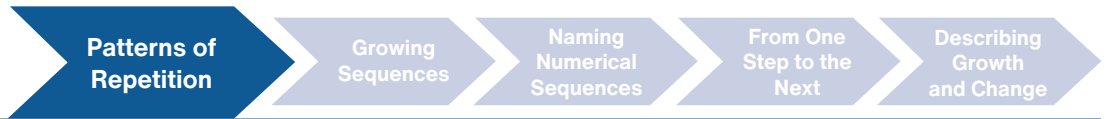
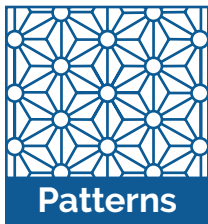
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Patterns in the Physical World

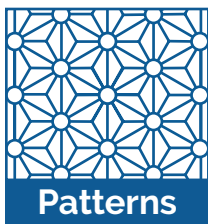
Goal: Learn more about patterns in the world.

1. With your partner, take turns reading a section from the text, *Patterns in the Physical World*.
2. After your turn reading each section, select a strategic action from the *Clarifying Bookmark I* and talk with your partner. Then, your partner will respond to your ideas, possibly choosing another action from the *Clarifying Bookmark I*.
3. Repeat reading and sharing ideas using the *Clarifying Bookmark I*.
4. As you read with your partner, keep a running list of examples of patterns in your everyday world, and also consider the focus question: *Why are geometrical patterns so common in the world?*



Clarifying Bookmark I

What you can do	What you can say
I am going to think about what the selected text may mean.	<i>I'm not sure what this is about, but I think it means ...</i>
	<i>This part is interesting, and I think it means ...</i>
	<i>After rereading this part, I think it may mean ...</i>
I am going to summarize my understanding so far.	<i>What I understand about this reading so far is ...</i>
	<i>I can summarize this part by saying ...</i>
	<i>The main points of this section are ...</i>



Patterns

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Patterns in the Physical World

As you read, consider the focus question, why are geometrical patterns so common in the world?

Patterns in the Natural World

Patterns occur in the natural world because of how plants and animals grow. For example, sunflowers grow into round flowers with seeds in the middle. As more and more seeds grow from the center outward, the seeds form patterns in circles.

Many other organisms have these repeated patterns, such as seashells and pinecones. Other animals, such as zebras and tigers, have developed patterns on their skin to either hide from predators or to hunt other animals.

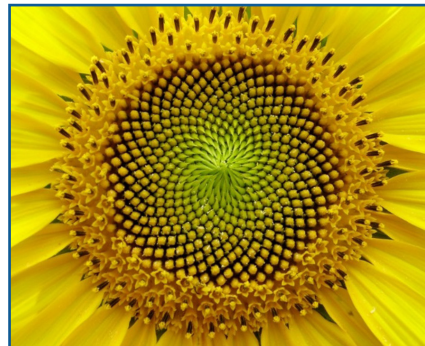
Patterns in the Built Environment

When people design buildings, they also use patterns. These patterns can serve both decorative and functional purposes. For example, glass windows look nice and let in light and air, and tile floors are durable and easy to clean.

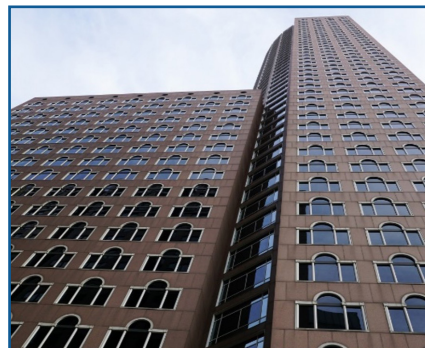
These patterns can make buildings more comfortable places to live and work. Repeating patterns also makes buildings easier to construct.

Patterns that You Can Wear

Another place to find patterns is in the clothing that people wear. Many clothes are knit or woven out of thread or yarn. It often is visually appealing to make patterns using different kinds of thread and shapes that are repeated over and over again. Warmer clothing like sweaters are also knit with repeated loops, which makes the clothing strong and flexible, but also beautiful.



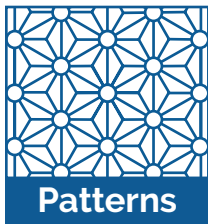
What patterns do you see in this sunflower?



What patterns do you notice in this building?



What patterns do you see in this fabric?



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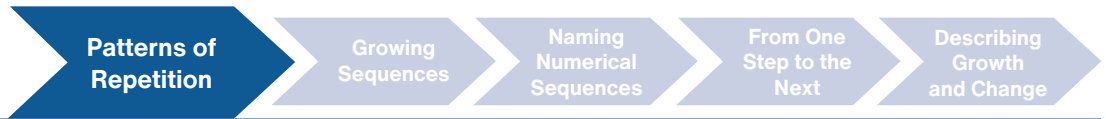


Describe and Order

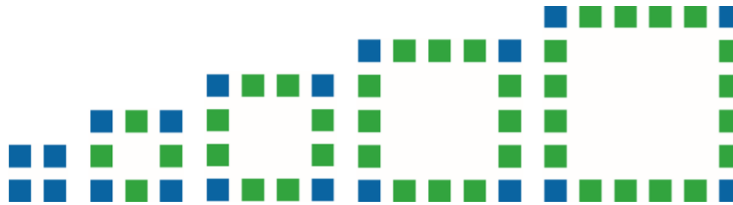
Goal: Describe different shapes and put them in order.

1. Four students will receive four different cards. They will each describe what is on their card to the whole class, giving as much detail as possible and without showing each other the card.
2. As a group, they will decide in what order the cards should go, without showing the cards.
3. **Listen** to how the students describe their cards. Consider what elements of their descriptions were most helpful as they put the cards in order.
4. **Discuss as a group:** What are the key elements of describing the cards to put them in order?



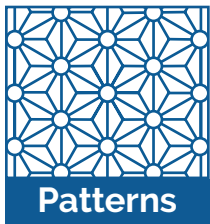


Daily Writing Prompts: Patterns of Repetition



1. **Draft** a description for how these shapes from the *Describe and Order* activity are changing:

2. **Review your description above.** What are the strengths of your description?



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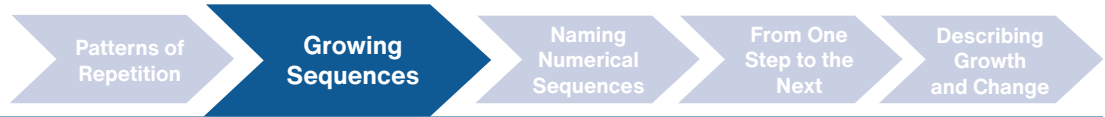
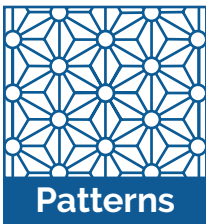
Describing
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Introducing Sequences

Goal: Learn more about sequences.

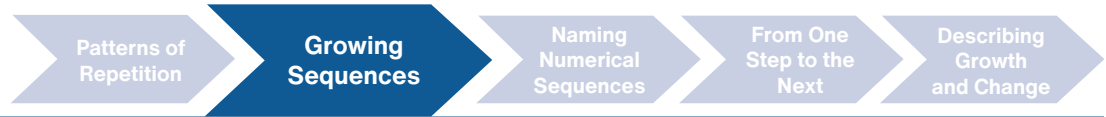
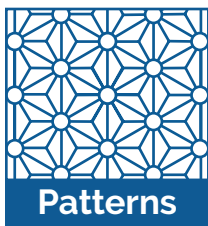
1. With your partner, take turns reading a section from the text, *Introducing Sequences*.
2. After your turn reading each section, select a strategic action from the *Clarifying Bookmark II* and talk with your partner. Then, your partner will respond to your ideas, possibly choosing another action from the *Clarifying Bookmark II*.
3. Repeat reading and sharing ideas using the *Clarifying Bookmark II*.
4. As you read with your partner, consider the focus question: *How can you create and extend sequences?*



Clarifying Bookmark II

What you can do	What you can say
I am going to think about what the selected text may mean.	<i>I'm not sure what this is about, but I think it means ...</i>
	<i>This part is interesting, and I think it means ...</i>
	<i>After rereading this part, I think it may mean ...</i>
I am going to summarize my understanding so far.	<i>What I understand about this reading so far is ...</i>
	<i>I can summarize this part by saying ...</i>
	<i>The main points of this section are ...</i>

What you can do	What you can say
I am going to use my prior knowledge to help me understand.	<i>I know something about this from ...</i>
	<i>I have read or heard about this when ...</i>
	<i>I don't understand the section, but I do recognize ...</i>
I am going to apply related concepts and/or readings.	<i>One reading/idea I have encountered before that relates to this is ...</i>
	<i>We learned about this idea/concept when we studied ...</i>
	<i>This concept/idea is related to ...</i>



Introducing Sequences

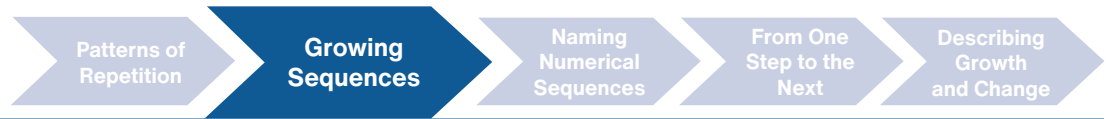
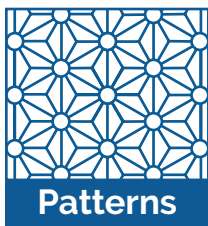
As you read, consider the focus question: How can you create and extend sequences?

A sequence is a list that has an order. The individual elements of a sequence are called terms. Sequences can be geometrical, numerical, or even just the days of the week. The key thing about a sequence is that it has an order in which ideas are expected to come. When the order is consistent and regular, it becomes something that is predictable and extendable.

For example, some schools have “block” schedules, so that not every class meets every day of the week. You might meet with your math class on Mondays, Wednesdays, and a shorter period on Friday. For science class, you might have Tuesdays, Thursdays, and a shorter period on Fridays. A similar pattern is typical of college classes, which tend to be longer and only meet two or three times a week (e.g., Tuesday and Thursday or Monday, Wednesday, and Friday).

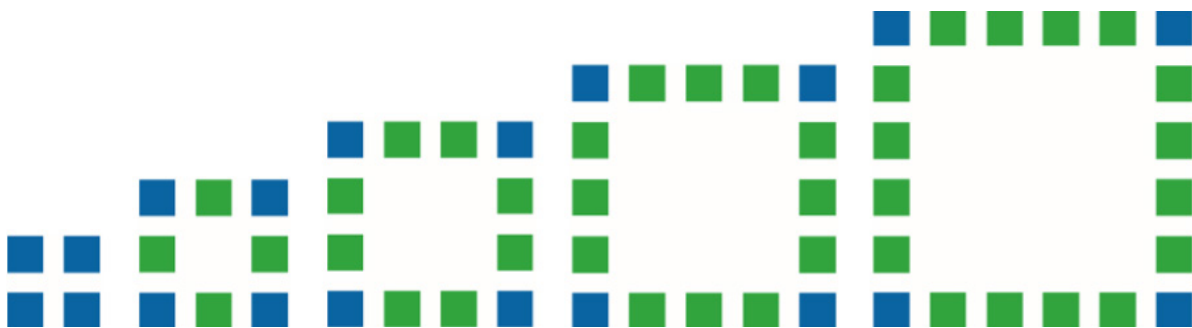
Within such schedules, you can make predictions about what is coming next, and there are regular patterns when you look at the dates of classes. For example, if you look just at the Monday classes, you might in a month meet on the 2nd, 9th, 16th, and so on. When putting together a calendar, the weekly rhythm forms a regular pattern.

When dealing with sequences, it is helpful to look at both the individual terms and the sequence as a whole and how it is changing. You will learn how to look at many different kinds of sequences, how they are growing, and how to make predictions about what comes next.

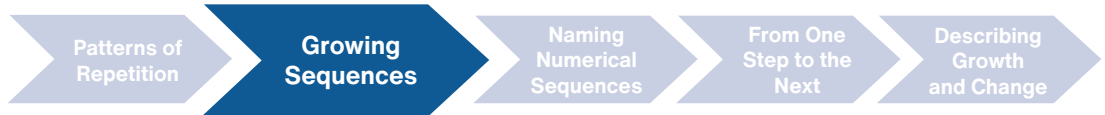
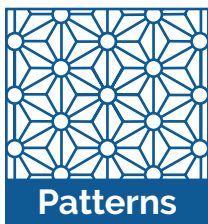


Reading in Four Voices

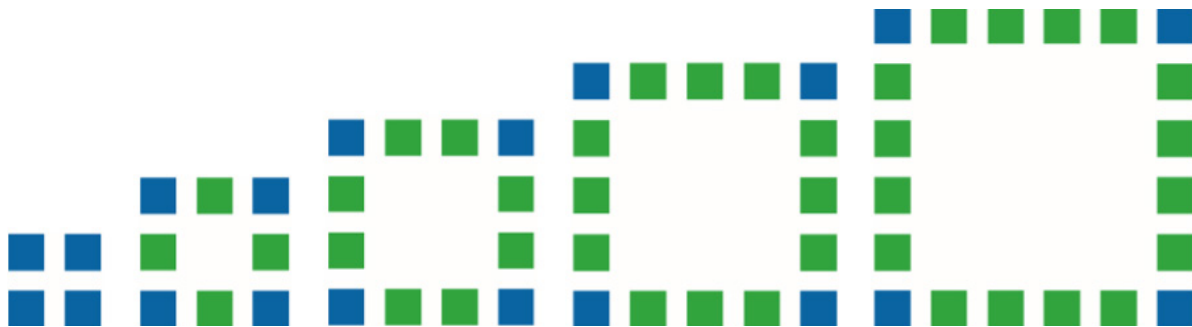
Goal: Connect parts of a written description to the growth of a sequence.



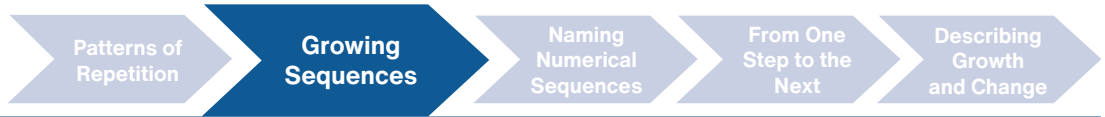
1. You will work in a group of four. Each member of your group will choose a style of font when reading the descriptions: plain, **bold**, underlined, or *italic*.
2. **Annotate.** There are two descriptions, each on its own page. Each student will read their style of font for each description. As you read your font style, annotate the picture to mark what you hear in the description. Your annotations could include arrows, circling, or other markers to make connections between the description and the shapes. All group members will annotate the same picture.
3. Repeat this process for the second descriptions.
4. After you have read both descriptions and annotated both pictures, decide which one you think is better for the purposes of understanding how the squares will grow and for making predictions beyond, such as what will be the tenth term in the growing sequence.
5. Then, make further edits to the description you chose to improve it and to make it easier to use for making predictions on how the squares will grow in the sequence.



Reading in Four Voices: Description 1



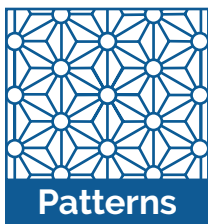
Each of the images has four blue squares in the corners. The other squares are in between on the sides and they are green. The first shape has no green squares, while each new shape has more and more green squares. The second shape has a total of four green squares, **one on each of the sides**. Then, the next shape has four more green squares, for a total of eight, with two on each side.



Reading in Four Voices: Description 2



The shapes are built out of green squares connected to some blue squares. **As you go around the frame clockwise, there are four pieces, each beginning with a blue square and ending with some green squares.** The first shape has just four blue squares. **The second shape uses a blue square and one green square four times going around to make the frame.** Each frame has the same number of total squares on each of its four sides, the same as the term number—one of these is blue and the rest are green.



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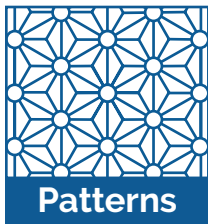
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Describe and Order: Expert Group

Goal: Describe terms of a sequence and put them in order.

1. You will work in a group of three or four students. Each group member will receive a card. **Do not show each other your cards.**
2. **Describe.** Describe what is on your card to the other students in your group, giving as much detail as possible and without showing your card.
3. **Order.** Make sure each person in the group takes a turn to describe. Then, decide as a group in what order the cards should go, without showing each other your cards.
4. As you reach an agreement, place the cards face down on the table in the order you think they belong. Each group has a total of five cards, so there may be an additional card or cards. If so, try to describe what is on that card before you look at it together.
5. **Reveal.** Once everyone has agreed, flip the cards over to see if the order you have selected makes sense.



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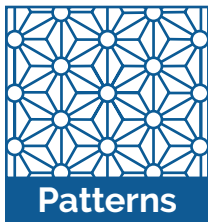
Describing
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Structure and Regularity Guidance Card

Goal: Describe the structure of patterns and what is regular in them.

1. You will work with a partner now that the sequence has been revealed.
2. Take turns talking with your partner, selecting actions from the left side of the below card, and then saying a particular expression from the right side.
3. Try different ways of talking about the pattern. Reach an agreement with your partner about what you think is the most effective way to present the structure and regularity that you see.
4. Be prepared to share with the other pair in your original group.



Patterns

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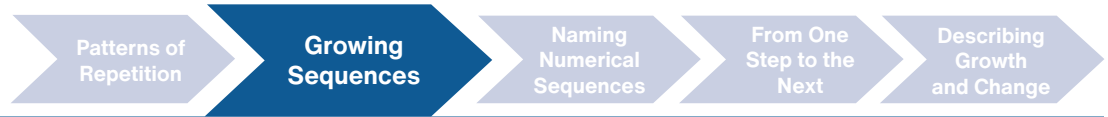
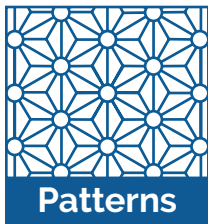
Describing Growth and Change



Structure and Regularity Guidance Card

What you can do	What you can say
Point out what is the same or different.	<i>Both of these ... have ...</i>
	<i>When I look at ..., this ... has more/less ...</i>
	<i>Unlike ..., this ... has ...</i>
Put into groups and take groups apart.	<i>If I put together ..., I get ...</i>
	<i>If I look at ..., I see ...</i>
	<i>If I take apart ..., I have ...</i>
Find and describe what repeats.	<i>Something that I see repeating is ...</i>
	<i>If I keep doing ..., I will get ...</i>
	<i>A shortcut for repeating this is ...</i>

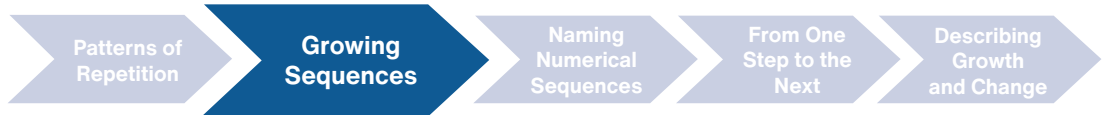
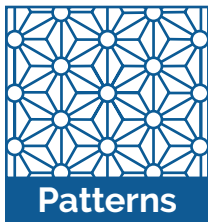




Individual Matrix

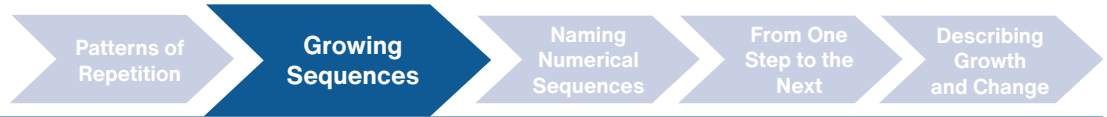
Goal: Detail what you have learned about sequences from your work with your expert group.

- 1. Collect your thoughts independently.** Respond to each question below by thinking through what you've learned through the activities that you completed with your expert group.
- 2. Share and discuss.** Take turns sharing your drafted description and your ideas for each question. Each student should share their responses while the others listen attentively.
- 3. Record.** As a group, reach a consensus and decide on a bulleted list for your drafted description of your sequence and write it on the *Expert Matrix* next to the prompt in the column that corresponds with your topic. Then, repeat this process for each question until you have shared your ideas for each question and recorded responses on the *Expert Matrix*.



Individual Matrix

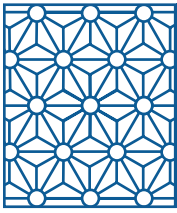
Prompt	My Response
Draft a bulleted list that describes the first three terms of the sequence. In your base group, your group members will draw your sequence based on your description.	
What are the numerical terms of the sequence?	
How is the sequence changing?	
What will the tenth term of the sequence be?	



Expert Matrix

Goal: Share what you learned about sequences from your work with your expert group.

- 1. Record.** You will use the *Expert Matrix* to record what your expert group agreed on for each question.
- 2. Describe.** In your base group, use your draft description to describe your sequence so each member of your base group can draw it.
- 3. Share.** Also in your base group, take turns sharing what you learned in your expert group. Each student should share their responses while the others listen and take notes. Make notes in the *Expert Matrix* on what you learn from the other groups.



Patterns

Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

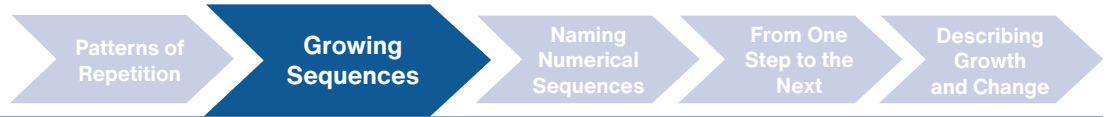
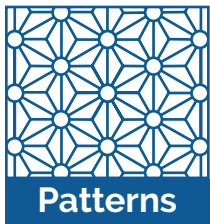
From One Step to the Next

Describing Change and Growth



Expert Matrix

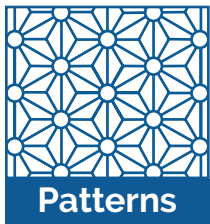
Prompt				
Sketch the first three terms of the sequence.				
What are the numerical terms of the sequence?				
How is the sequence changing?				
What will the tenth term of the sequence be?				



Daily Writing Prompt Day 2: Growing Sequences

1. Review the list of bullets that you used to describe your sequence. **Now that you have tested your description with your peers, make notes on what you add or take away from your description.**

2. Below, draft a new description of your sequence using what you have learned through sharing your description and listening to other descriptions.



Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

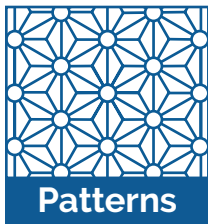
Describing Growth and Change



Counting Handshakes

Goal: Model and solve problems resulting in a sequence of answers.

1. Independently, read the question: *Starting with two people, how many handshakes can a group of people make so that everyone shakes hands with everyone else?*
2. Next, independently draw out how you would determine the number of handshakes that a group of people would make so that everyone shakes hands with everyone else if:
 - There are three people in a group
 - There are four people in a group
3. With a partner, discuss how you determined the number of handshakes that a group of 3 people and a group of 4 people would make. Listen to each other and compare your drawings and the number of handshakes for each group.
4. Then, watch as five students act out shaking hands. Before they begin, discuss with your partner how you will count handshakes systematically.
5. As the five students act out shaking hands, count and represent their handshakes. Have a full-group discussion about the number of handshakes that group of 5 people would make and how it is related to your representation.
6. Then, work with your partner to come up with how many handshakes there would be for a **group of 10 people**. Finally, write out instructions or a **systematic “rule” about the number of people in a group and the number of handshakes so everyone shakes hands with everyone else.**



Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

Describing Growth and Change



Step It Up

Goal: Co-create and name numerical sequences.

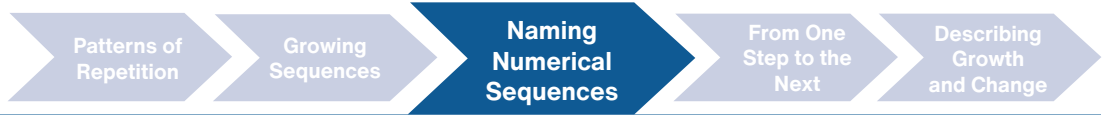
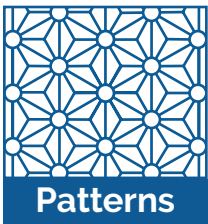
1. With your partner, determine who will be Student A and who will be Student B. Student A will begin by suggesting an operation, such as “add 4” or “multiply by 3.”
2. Student B will then write the formula and terms using the suggested operation.
3. Take turns suggesting operations and completing the formula and terms.
4. **Discuss with your partner.** When you and your partner have completed all steps, discuss your sequences and what you learned:

Consider the following questions as you work:

- *Is there another way you could have ended up with the same sequence?*
- *That is, can you find another set of operations that would give you the same numerical sequence?*

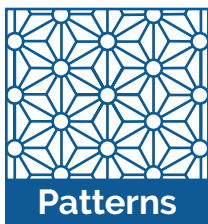
5. **Exchange.** Exchange your final sequence with another pair. See if they can come up with the formula for your sequence.





Step It Up Recording Sheet

Student	Formula	Terms	Operation
A	N	1, 2, 3, 4, 5, ...	
B			
A			
B			
A			
B			



Patterns of
Repetition

Growing
Sequences

**Naming
Numerical
Sequences**

From One
Step to the
Next

Describing
Growth
and Change

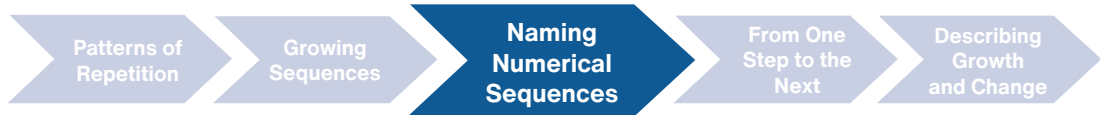
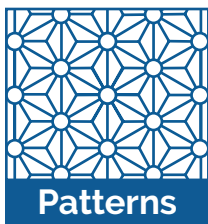


Read and Connect: Formulas

Goal: Connect elements of a verbal description to a formula.

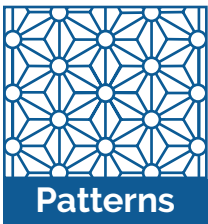
1. You will work in a group of four. Together, you will revisit the pattern and descriptions from a previous task.
2. **Examine.** This time, each student will receive a new card with a formula written on it. Look at it carefully and see if you can identify parts.
3. **Read and Listen.** Take turns reading a description from below out loud to your group. Each student will review their formula while one student reads. Note any connections between the description read and the formulas.
4. **Connect.** After each member has read a description, discuss as a group whether you think your formula could fit the description. Connect the parts of the description to the parts of the formula. Make sure they all agree.
5. **Review and Evaluate.** As you look back at the collection of four descriptions and four formulas, do the connections help you to change your mind?
6. **Generate and Check.** Working individually or with a partner using a Step it Up process, construct the formulas and compute the first five terms. Check that these terms match the numerical terms of the original sequence. Find statements that you can make about the differences for each sequence.





Read and Connect

Verbal description	Formula
<p>Each of the images has four blue squares in the corners. The other squares are in between on the sides, and they are green. The first shape has no green squares, while each new shape has more and more green squares. The second shape has a total of four green squares, one on each of the sides. Then, the next shape has four more green squares, for a total of eight, with two on each side.</p>	
<p>Each of the images is the outside of a square, made with squares. On each of the four sides of the square frame, there are the same number of blue and green squares. The first shape has two blue squares on each of the four sides. The second shape has two blue squares and one green square in the middle. Each shape has more and more green squares.</p>	
<p>The shapes are built out of green squares connected to some blue squares. As you go around the frame clockwise, there are four pieces, each beginning with a blue square and ending with some green squares. The first shape has just four blue squares. The second shape uses a blue square and one green square four times going around to make the frame. Each frame has the same number of total squares on each of its four sides, the same as the term number—one of these is blue and the rest are green.</p>	
<p>Each shape is the outside edge of a square, kind of like a picture frame. The first shape is a 2x2 square. The second shape is a 3x3 square with the inside square that is 1x1 removed. The next shape is 4x4 with a 2x2 square inside removed. With each shape, the overall square gets bigger, and so does the smaller square inside that is missing.</p>	



Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

Describing Growth and Change



Daily Writing Prompt Day 3: Naming Numerical Sequences

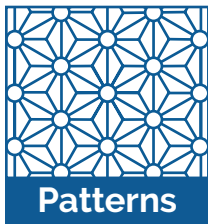
Looking back at the different descriptions of the growing patterns, what did you find the most useful and effective? How would you apply this to your own descriptions?

You are given the numerical sequence:

5, 8, 11, 14, 17, ...

How would you find a formula that matches this sequence?





Patterns of
Repetition

Growing
Sequences

Naming
Numerical
Sequences

**From One Step
to the Next**

Describing
Growth
and Change

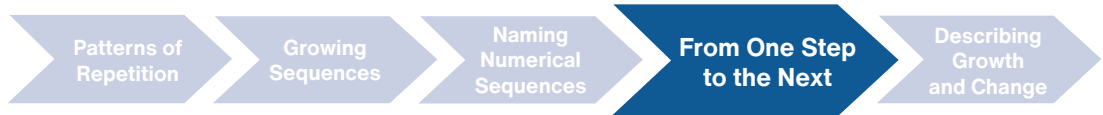
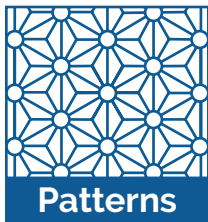


Patterns of Growth

Goal: Learn more about sequences and differences.

Reading with a Partner

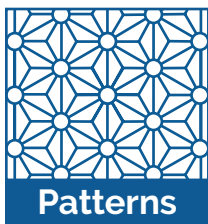
1. With your partner, take turns reading a section about sequences and differences from the *Patterns of Growth* text below.
2. After your turn reading each section, select a strategic action from one of the *Clarifying Bookmark II* and share it with your partner. Then, your partner responds to your ideas, possibly choosing another action from the *Clarifying Bookmark II*.
3. Repeat reading and sharing ideas using the *Clarifying Bookmark II*.
4. As you read the text with your partner, make sure to discuss the focus question in each section.



Clarifying Bookmark II

What you can do	What you can say
I am going to think about what the selected text may mean.	<i>I'm not sure what this is about, but I think it means ...</i>
	<i>This part is interesting, and I think it means ...</i>
	<i>After rereading this part, I think it may mean ...</i>
I am going to summarize my understanding so far.	<i>What I understand about this reading so far is ...</i>
	<i>I can summarize this part by saying ...</i>
	<i>The main points of this section are ...</i>

What you can do	What you can say
I am going to use my prior knowledge to help me understand.	<i>I know something about this from ...</i>
	<i>I have read or heard about this when ...</i>
	<i>I don't understand the section, but I do recognize ...</i>
I am going to apply related concepts and/or readings.	<i>One reading/idea I have encountered before that relates to this is ...</i>
	<i>We learned about this idea/concept when we studied ...</i>
	<i>This concept/idea is related to ...</i>



Patterns

Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

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Patterns of Growth

Different Differences

How do different sequences have different differences?

One useful tool for understanding numerical sequences is to look for patterns of change or growth. For any sequence, it is always possible to look at how it is changing, either through the operation of addition/subtraction or multiplication/division. But sometimes it takes more terms before a pattern is fully set.

For example, for the three illustrated sequences on the right, there are two numerical terms that all sequences have in common: 1 and 9. All three sequences are of the form 1, __, 9, __, Yet all three sequences grow very differently. Sequence A grows with a constant increase of +4, over and over again. Sequence B grows with differences that are changing: 3, 5, 7, ...

A: 1, 5, 9, 13, 17 ...

B: 1, 4, 9, 16, 25 ...

C: 1, 3, 9, 27, 81 ...

Discuss with your partner and come to an agreement: How would you compare sequence C to the others?

Making the Difference

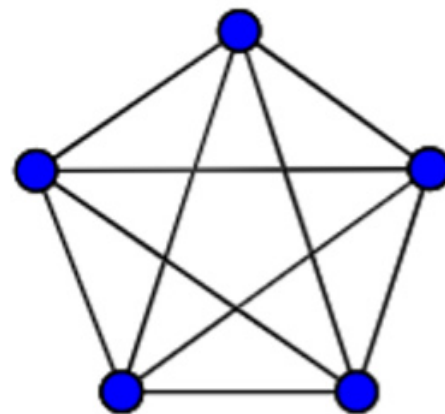
How does looking at differences as a sequence help to describe patterns?

In solving the handshake problem, it is possible to construct the sequence of answers as:

1, 3, 6, 10, ...

Looking at the differences in this sequence gives: 2, 3, 4, ..., which is a regular pattern. This difference sequence makes sense in the context of the scenario in part because you can think about what happens when an additional person joins the group—some new handshakes will be necessary.

Specifically, everyone who is already in the group will need to shake hands with the new person. The differences are growing in a regular way.



This graph represents all handshakes possible between five individuals. The blue dots each represent a person. The lines between the dots each represent a handshake.

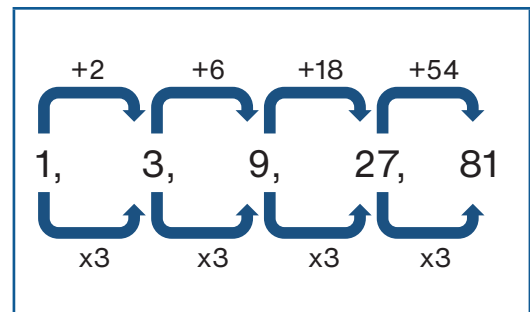
Discuss with your partner: When you look at the handshake sequence, which of the sequences A, B, and C do you think it is the most similar to? Why?



Changing Operations and Operations of Change

How can other operations describe how sequences grow and change?

In examining sequence C, however, looking at the differences reveals an interesting pattern. Since C is 1, 3, 9, 27, ..., the differences are 2, 6, 18, These differences are not growing by the same amount each time. In this case, rather than look at differences, which come from adding or subtracting, we might want to look at another operation that can connect the terms of our sequence—multiplication.

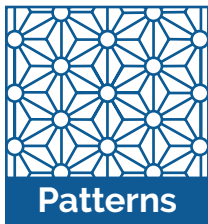


Finding this rule means looking for a multiplier or factor that takes one term to the next. For the original sequence, the multiplier is 3—each term is the previous term multiplied by 3.

There may be other sequences where you can use a multiplicative approach. For example, consider the sequence:

$$5, 20, 80, 320, 1280, \dots$$

Discuss with your partner: What do you notice about how this sequence is changing multiplicatively?



Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

Describing Growth and Change



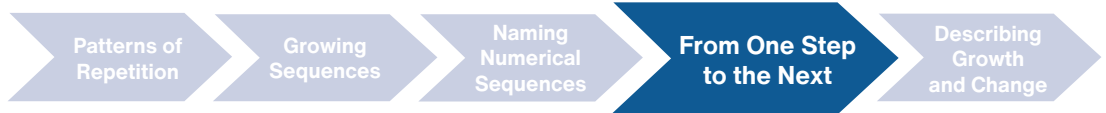
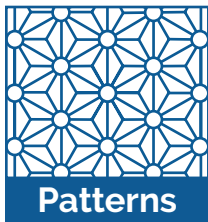
Problem-Solving with a Partner

1. Select one of the following two problems to try and solve with your partner:

Dominos are twice as tall as they are wide. Suppose you want to cover a rectangular space that is 2×5 in area. How many ways can you completely fill that space with dominos? What about a space that is 2×10 in area? How large a space would you need to have more than 100 ways to stack your dominos?

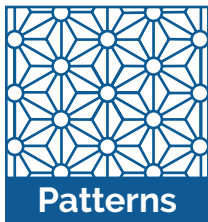
There are different ways to climb stairs. Someone is trying to climb a fixed number of steps by going up either one step or two steps each time. How many ways are there to climb a staircase that is five steps tall? What if instead there were ten steps total? How tall a staircase would enable more than 100 ways to climb it?

2. Use the *Bookmark to Understand the Problem* below to discuss the problem with a partner. Then, your partner responds to your ideas, possibly choosing another action from the *Bookmark to Understand the Problem*.
3. Find a new partner who solved a different problem than you did. Compare and contrast these two problems with one another. As you discuss, consider what pattern of growth you think this is most like: A, B, or C? Why?
4. After working with your partner, work independently to respond to the *Daily Writing Prompt* on the next page.



Bookmark to Understand the Problem

What you can do	What you can say
Identify what the problem is asking.	<i>The unknown in this problem is ...</i>
	<i>The units of the unknown are ...</i>
	<i>Reasonable values for the unknown would be ...</i>
Identify the given data and constraints.	<i>The variables or quantities in this problem are ...</i>
	<i>The values given in the problem are ...</i>
	<i>This problem assumes that ...</i>
Draw a picture or model to represent the problem in a different way.	<i>I can show this problem by ...</i>
	<i>A model that represents this problem is ...</i>
	<i>I can represent this part of the problem with ...</i>



Patterns

Patterns of
Repetition

Growing
Sequences

Naming
Numerical
Sequences

**From One Step
to the Next**

Describing
Growth
and Change



Daily Writing Prompt Day 4: From One Step to the Next

Look at the two patterns that have emerged from the two story problems. Write an individual response about whether you think this pattern is most similar to sequences A, B, or C from the reading. Specifically:

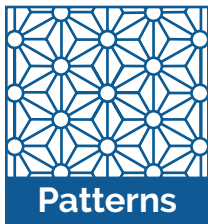
A: 1, 5, 9, 13, 17, ...

B: 1, 4, 9, 16, 25, ...

C: 1, 3, 9, 27, 81, ...

Give detailed reasons for your selection.





Patterns of
Repetition

Growing
Sequences

Naming
Numerical
Sequences

From One Step
to the Next

Describing
Growth
and Change



Flipping Triangles

Goal: Explore and generalize patterns.

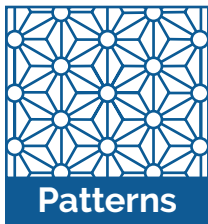
Consider a sequence of triangles which are given by the following description:

The first triangle has two rows, with one object in the first row and two objects in the second row.

The second triangle has an additional third row, in which there are three objects.

Each new triangle has an additional row which has the same number of objects as the total number of rows.

1. Work with your group and investigate the following question: *For each triangle in the sequence, what is the fewest number of objects you need to move so that the triangle is pointing “down”—in the opposite direction?* Note that the position of the triangle does not matter, so long as there is one object in the bottom row and the most objects in the top row.
2. Use the *Bookmark for Planning the Solution* below to discuss the problem with a partner. Then, your partner responds to your ideas, possibly choosing another action from the *Bookmark for Planning the Solution*.
3. Identify and describe a pattern that will enable you to generalize to a triangle with any number of rows.



Patterns

Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

Describing Growth and Change



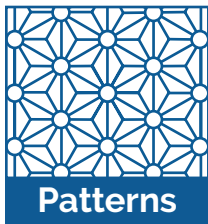
Counting Paths

Goal: Explore and generalize patterns.

Consider a city grid with a starting point in the northwest (or upper left) and an endpoint in the southeast (lower right). Suppose a walker begins in the northwest and wants to walk to the southeast endpoint without going backwards or creating loops. The starting point is two blocks north of the endpoint. The east-west distance between the starting and end points can vary, as a whole number of city blocks. How many ways are there to walk, for different widths, starting with 0 blocks?

1. Work with your group and investigate the following question: *For each grid in the sequence, what is the total number of different possible walks from start to finish?*
2. Use the *Bookmark for Planning the Solution* below to discuss the problem with a partner. Then, your partner responds to your ideas, possibly choosing another action from the *Bookmark for Planning the Solution*.
3. Identify and describe a pattern that will enable you to generalize the ways to walk for any number of widths.





Patterns

Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

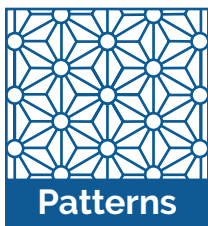
Describing Growth and Change



Bookmark for Planning a Solution

What you can do	What you can say
Think of a similar or related problem you have solved.	<i>A problem similar to this is ...</i>
	<i>I solved another problem like this by ...</i>
	<i>This problem is similar to ... because ...</i>
Find a tool that will help you.	<i>I can make a table with ...</i>
	<i>A graph of ...</i>
	<i>An equation to represent ...</i>
Examine an example or case.	<i>If I substitute the value ... for ...</i>
	<i>One special case of this problem is when ...</i>
	<i>An example of this would be ...</i>





Patterns of
Repetition

Growing
Sequences

Naming
Numerical
Sequences

From One Step
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Describing
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Writing Extension Activity

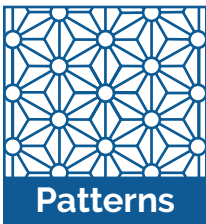
Goal: Develop descriptions and connections across solutions to problems that are sequences.

Over the course of this module, you have solved multiple problems that involve sequences as solutions. These problems have included patterns given as the growth of objects, as well as patterns and sequences that result as the solutions to story problems of various types, including:

- Counting Handshakes
- Climbing Stairs
- Packing Dominos
- Flipping Triangles
- Counting Walks

Now is your opportunity to demonstrate what you individually understand by completing an extended piece of writing that explains your approach and solution to the problem, as well as how to further extend the patterns that you have unpacked and discovered. You will put together the various mathematical tools that you have developed throughout the module, including:

- Written descriptions of scenarios, shapes, numbers, and changes.
- Connections between symbolic formulas, numerical values, and real-world scenarios or situations.
- Methods for extending patterns beyond the given terms, whether through rules or a formula.
- Explicit comparisons between two different patterns with attention to the kind of growth that it represents.



Patterns

Patterns of Repetition

Growing Sequences

Naming Numerical Sequences

From One Step to the Next

Describing Growth and Change



Writing Extension Activity Rubric

Category	Indicators of High Quality Work	Strengths	Areas to Improve or Revise
Content	<ul style="list-style-type: none"> Written descriptions provide detail that enables connections about numbers, shapes, and changes. Analysis of changes in the sequence of answers are connected to the meanings in real-world scenarios. Information or an approach is developed to extend the pattern. Explanations explicitly compare and contrast different patterns in terms of growth and meaning. 		
Design	<ul style="list-style-type: none"> Use of visual images and color is effective for adding to the meaning communicated. Elements combine to show or highlight connections. 		





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