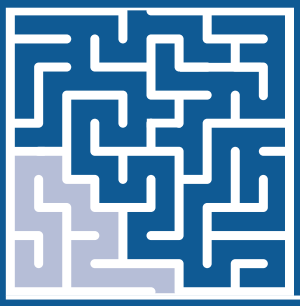
National Research & Development Center to Improve
EDUCATION FOR SECONDARY ENGLISH LEARNERS
WestEd



STUDENT MATERIALS

Equivalence & Transformation:
Can you get from here to there?

Student Name: _____

Class: _____

Module Overview

In this module, you will explore idea about equivalence in mathematics—when different objects are the same in some way. You will examine equivalence across contexts which will build your understanding of what equivalence can mean in different cases.

Mazes and Mean → **Slides and Codes** → **Perspective Geometry** → **Extending Equivalence**

Can You Get There from Here? (And Back!)

Goal: Identify and apply properties that determine equivalence.

1. **Watch the video.** The video introduces ideas about mazes, moves, and equivalent pieces.
2. **Explain what positions in the maze are equivalent** and how you know. Discuss with your partner.
3. **Make a maze** using the dot paper below, make a maze and label some points. Exchange papers with a classmate and try to separate the maze into different connected components.

Can You Get There from Here? (And Back!)

Compare and Contrast Matrix: Mazes

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, read each question on the *Compare and Contrast Matrix* to understand what the question is asking and how you could fill out a response related to the **maze activity**. Alternate reading each question with your partner.
2. Reach a consensus on what you both will write in the column about mazes. Then write your responses in the column.
3. Be prepared to share your responses with the whole class.
4. Later in this module, you will return to this handout and complete other columns in the
5. *Compare and Contrast Matrix* after learning about equivalence in other contexts.

Compare and Contrast Matrix: Mazes

Prompt	Mazes	Mean as Balance	Slide Puzzles	Alphabetic Code	Perspective Geometry
What different types of objects did you compare and connect?					
What moves can you make to maintain equivalence?					

Prompt	Mazes	Mean as Balance	Slide Puzzles	Alphabetic Code	Perspective Geometry
What moves can you not make, and why?					
What other ways could you check whether two objects are equivalent?					

Novel Ideas Only

Goal: Generate ideas about the word "balance."

Today you will explore the idea of "equivalence" and how it is connected to the idea of what is the "balance." You will explore how moves or transformations are related to the idea of equivalence by exploring the topic of unit rates.

Part 1: Small Group Brainstorming

1. Take turns sharing an idea in response to the prompt, "When I hear the word 'balance' I think of ..." Each time, someone should read the prompt out loud, then offer an idea.
2. After one student offers an idea, another student will repeat that idea out loud and all students in your small group will write down the exact same information below. Everyone in the group should have the exact same information on their papers.
3. Continue sharing ideas, repeating them out loud and writing them down. When time is up, draw a line under the last idea written down in your list.

Part 2: Full Class Sharing-Out

4. Groups will all stand up. One student from each group will read the prompt out loud and then list off their group's ideas.
5. The other groups must listen carefully as groups share. When your group shares, do not repeat any of the ideas that have already been shared. You can keep track of the ideas from your list that others share by crossing off the lightbulb if another group has your same idea.
6. After a group has shared, they sit down. Group members must still keep listening, however, as they may individually add a few new ideas you really like to your list.

Novel Ideas Only

When I hear the word "balance," I think of ...

Think Pair Share

Goal: Discuss what balance means in terms of rocks on a seesaw and connect to the concept of equivalence.

1. **Think.** Jot down ideas around the prompt that your teacher will project.
2. **Pair.** With your partner, take turns sharing your ideas you wrote.
3. **Share.** Be prepared to share what your partner said during your discussion.

Human Number Line

Goal: Experience how numerical data is distributed and make connections to ideas of equivalence.

1. After your teacher ask you the question for this task, write your response on a sticky note.
2. Move to stand in the location that corresponded to your written response. Together, you and your classmates will form a **number line**.
3. Post your sticky note on the board. The posted sticky notes will form a **dot plot**.

Step it Up: Balance

Goal: Co-create dot plots to highlight the point of balance.

In this activity, you will work with your partner to transform a variety of dot plots to make it easier to see the point of balance. Your goal is to transform the dot plot in a way that will not change the overall point of balance.

1. With your partner, determine who will be Student A and who will be Student B. Student A will begin by suggesting a move to the dot plot.
2. Student B will then draw the new dot plot.
3. **Take turns** suggesting changes and completing the dot plot, seeking to make it easier to see the point of balance.
4. **Discuss your dot plots with your partner.** When you and your partner have completed all steps, discuss your dot plots and what you learned.

Consider the following questions as you work:

- Is there another way you could have ended up with the same dot plot?
- That is, is there another set of moves that would give you the same dot plot?

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Mean as Balance Step it Up Recording Sheet

Partner	Changes	Responses	Dot Plot
			1 2 3 4 5 6 7 8 9
A			1 2 3 4 5 6 7 8 9
B			1 2 3 4 5 6 7 8 9
A			1 2 3 4 5 6 7 8 9
B			1 2 3 4 5 6 7 8 9

Partner	Changes	Responses	Dot Plot
			1 2 3 4 5 6 7 8 9
A			1 2 3 4 5 6 7 8 9
B			1 2 3 4 5 6 7 8 9

Sort and Order

Goal: Connecting conceptions around mean with balancing conceptions as it relates to equivalence.

1. **Read.** The first student will take one card and read it to the group without showing them the card.
2. **Order.** The next student will take another card, read it, and place it above or below the first card, and share their reasoning about their placement in relation to the other cards.

   You may find the following language helpful as you work:

   - *I think this line plot matches this one because ...*
   - *This group of line plots all have the same ...*

3. **Discuss and Label.** Discuss with your group and come to a consensus on what label best represents the groups you have created. After all the cards have been placed on the table, discuss whether any cards need to be changed.

Compare and Contrast Matrix: Mean as Balance

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about mean as balance.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to means as balance. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about mean as balance. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

Daily Writing Prompt: Introducing Equivalence and Transformation

Draw three line plots that are not equivalent from the point of view of balance. Explain why they are not equivalent to each other.

List at least two connections between getting through mazes and balancing line plots.

Explore Slide Puzzles

Goal: Identify properties that determine equivalence.

1. **Watch the slide puzzles presentation through Part 1.** Talk with a partner
   - What are the moves possible in a slide puzzle?
   - Is it possible to solve the slide puzzle on slide [5]? Explain why.
2. **Watch the slide puzzles presentation through Part 2.** Discuss the following questions with a partner:
   - What is a solvable case of a slide puzzle?
   - Where do you see equivalence in the 2x2 slide puzzle?
   - Why is the slide puzzle on slide [16] not solvable? Explain why

Clarifying Bookmarks

Clarifying Bookmark I

What you can do	What you can say
I am going to think about what the selected text may mean.	*I'm not sure what this is about, but I think it means ...*
	This part is interesting and I think it means ...
	After rereading this part, I think it may mean ...
I am going to summarize my understanding so far.	*What I understand about this reading so far is ...*
	I can summarize this part by saying ...
	The main points of this section are ...

Clarifying Bookmark II

What you can do	What you can say
I am going to use my prior knowledge to help me understand.	*I know something about this from ...*
	I have read or heard about this when ...
	I don't understand the section, but I do recognize ...
I am going to apply related concepts and/or readings.	*One reading/idea I have encountered before that relates to this is ...*
	We learned about this idea/concept when we studied ...
	This concept/idea is related to ...

Clarifying Bookmark III

What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Compare and Contrast Matrix: Slide Puzzles

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about slide puzzles.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to slide puzzles. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about slide puzzles. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

Alphabetic Codes

Goal: Learn how alphabetic codes can be used to keep information private.

In this activity, you will review a screencast, and consider the guiding question, "How do you keep your information private?"

1. Watch the slides/presentation, *Alphabetic Codes.*
2. During the pauses, have a discussion with your partner.
3. Select an action from *Clarifying Bookmark III* and talk with your partner. Take turns sharing ideas and responding to each other's ideas.
4. Consider the questions:
   - How can you undo operations used to transform the alphabet?
   - Under what conditions are operations useful for making secret codes?
   - How do you keep your information private?"

Clarifying Bookmarks

Clarifying Bookmark I

What you can do	What you can say
I am going to think about what the selected text may mean.	*I'm not sure what this is about, but I think it means ...*
	This part is interesting and I think it means ...
	After rereading this part, I think it may mean ...
I am going to summarize my understanding so far.	*What I understand about this reading so far is ...*
	I can summarize this part by saying ...
	The main points of this section are ...

Clarifying Bookmark II

What you can do	What you can say
I am going to use my prior knowledge to help me understand.	*I know something about this from ...*
	I have read or heard about this when ...
	I don't understand the section, but I do recognize ...
I am going to apply related concepts and/or readings.	*One reading/idea I have encountered before that relates to this is ...*
	We learned about this idea/concept when we studied ...
	This concept/idea is related to ...

Clarifying Bookmark III

What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Partner Encryption

Goal: Create a code and represent it using a table.

Work with a partner to encode data.

1. Select a letter (or number) to be your multiplier. Then, use the table to write out the first half of the alphabet in code.
2. Tell your partner how you are using patterns to complete the table.
3. Then, your partner will finish the second half.

Partner Encoding Recording Sheet

		Multiplier		Responses
A	1			
B	2			
C	3			
D	4			
E	5			
F	6			
G	7			
H	8			
I	9			
J	10			
K	11			
L	12			
M	13			
N	14			
O	15			
P	16			
Q	17			
R	18			
S	19			
T	20			
U	21			
V	22			
W	23			
X	24			
Y	25			
Z	26=0			

Create, Exchange, and Assess

Goal: Explore alphabetic codes with a partner.

1. Work with a partner.
2. Draft a message of about 15-25 characters to encrypt.
3. Select a key with which to encrypt by picking a multiplier. Record your key in the Encryption Key table on the next page. Encrypt your message.
4. Exchange encrypted messages and keys with another pair. Try to figure out the original message.

Encryption Recording Sheet

Plain text	Plain number	Encrypted number	Encrypted letter

Sort and Label

Goal: Describe and label alphabetic codes.

1. **Read.** The first student will take one card and read it to the group without showing them the card.
2. **Group.** The next student will take another card, read it, and place it with the first card or start a new group, and share their reasoning about their placement in relation to the other cards.

   You may find the following language helpful as you work:

   - This card should be here because ...
   - I think this code is similar to or different from ... because ....
   - This card needs to be moved because ...

3. **Discuss.** After all the cards are placed on the table, discuss whether any cards need to be changed. Then, create a label for each group.

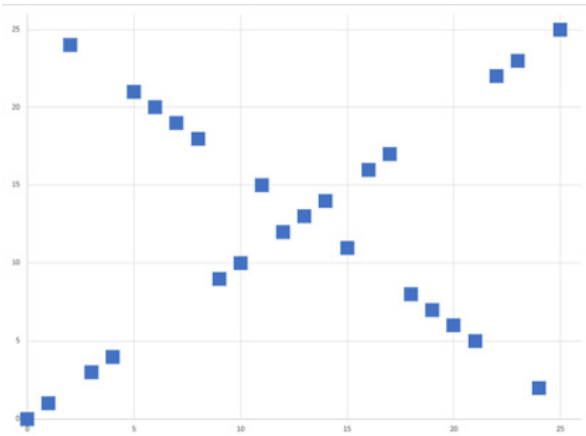
Compare and Contrast Matrix: Alphabetic Codes

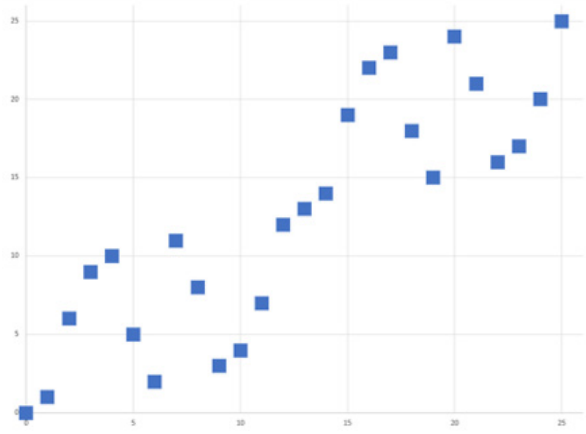
Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about alphabetic codes.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to alphabetic codes. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about alphabetic codes. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

Daily Writing Prompt: Alphabetic Codes

Review the visual representation of two different codes:





Which codes are decodable? Which codes can hide information well? Explain.

Perspective Geometry

Goal: Learn how to show perspective (3D) in the plane (2D).

In this activity, you will review a screencast, and consider the guiding question, "How do we show perspective in the plane?

1. Review the slides/presentation, *Perspective Geometry.*
2. Select an action from *Clarifying Bookmark III* and talk with your partner. Take turns sharing ideas and responding to each other's ideas.
3. Consider the questions.
   - How do vertical and horizontal lines look in the perspective plane, compared to what you are used to?
   - What could be a way to assign coordinates or equations to other points or lines in the perspective plane?

Clarifying Bookmarks

Clarifying Bookmark I

What you can do	What you can say
I am going to think about what the selected text may mean.	*I'm not sure what this is about, but I think it means ...*
	This part is interesting and I think it means ...
	After rereading this part, I think it may mean ...
I am going to summarize my understanding so far.	*What I understand about this reading so far is ...*
	I can summarize this part by saying ...
	The main points of this section are ...

Clarifying Bookmark II

What you can do	What you can say
I am going to use my prior knowledge to help me understand.	*I know something about this from ...*
	I have read or heard about this when ...
	I don't understand the section, but I do recognize ...
I am going to apply related concepts and/or readings.	*One reading/idea I have encountered before that relates to this is ...*
	We learned about this idea/concept when we studied ...
	This concept/idea is related to ...

Clarifying Bookmark III

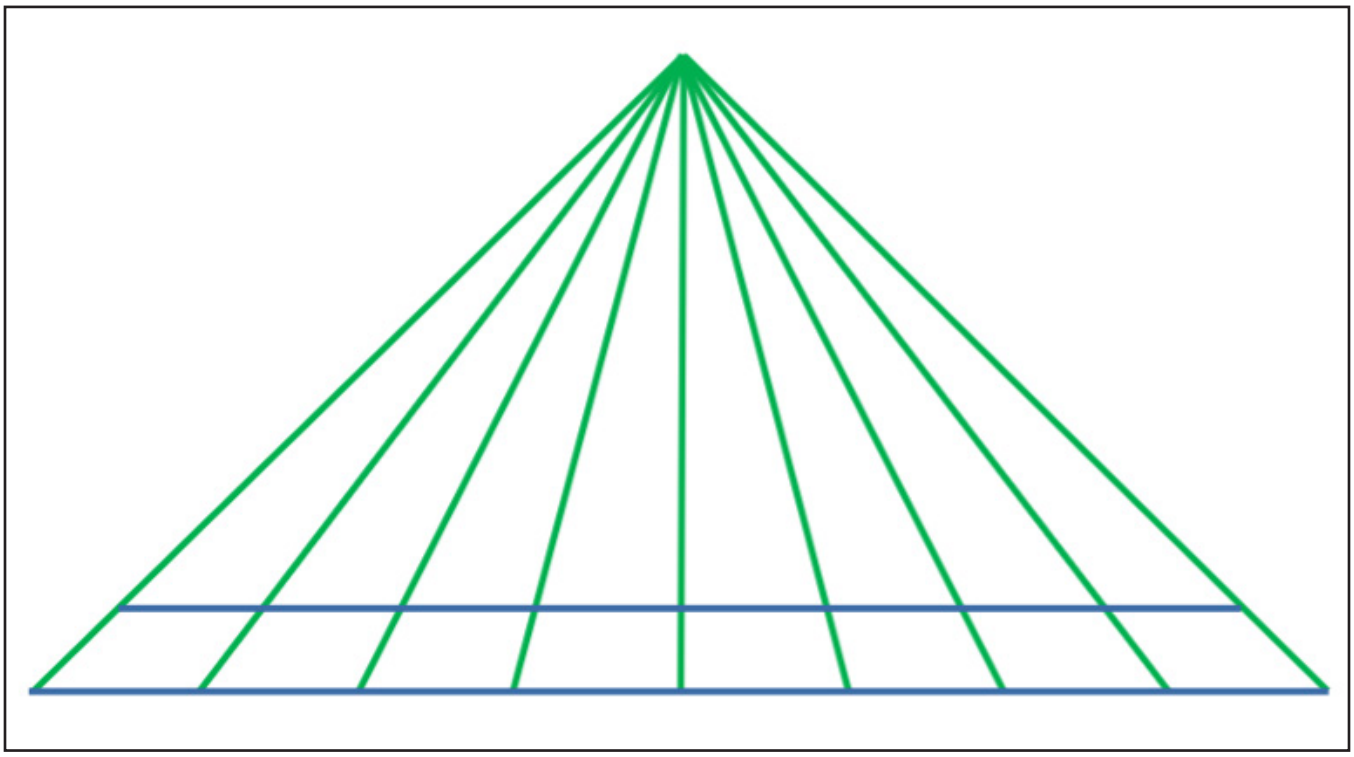
What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Step it Up

Goal: Co-construct new points and lines with coordinates and equations.

1. With your partner, determine who will be Student A and who will be Student B. Work on the shared plane in different colored pencils or pens.
2. Student A will begin by choosing to add something to the shared diagram. Student A will then record this in the table.
3. Student B will then try to add something new building on what A has just added wherever possible.
4. The objective is to add more and more elements to the perspective plane until it is as complete as the regular coordinate plane.
5. Consider the following questions:
    - How would you give different points in your grid coordinates in terms of x and y?
    - What lines can you draw that are not horizontal or vertical?
    - What slopes would those lines have?
    - What can you say about all lines with the same slope?

Step it Up: Perspective Geometry



Step it Up: Perspective Geometry

	What you will use	New element added
A		
B		
A		
B		
A		
B		

Compare and Contrast Matrix: Perspective Geometry

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix.* You will complete the column with questions about perspective geometry.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to perspective geometry. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about perspective geometry. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

Writing Extension Activity

Over the course of this module, you have explored multiple ways to define equivalence as related to transformations, including:

- Mazes
- Mean as balance
- Slide puzzles
- Alphabetic codes
- Perspective geometry

Now is your opportunity to demonstrate what you individually understand by completing an extended piece of writing that explains at least three of these ideas in great depth. In your written response, you will include the various ideas around equivalence that you have explored, including:

- A focus on the use of transformations or moves.
- Multiple representations, such as symbolic algebraic formulas, to connect different objects.
- Selection of key representatives of groups that are equivalent in some way.

Writing Extension Activity Rubric

Category	Indicators of High Quality Work	Strengths	Areas to Improve or Revise
Content	• Clearly unpacked examples illustrate key ideas and offer a general approach that could be extended to other examples. • General statements of fact and demonstrations of statements that are true or not true in different cases, with justifications for why these statements should be true. • Explanations of systems provide a rule or set of criteria for distinguishing different objects or proving that they are equivalent. • Explanations explicitly compare and contrast cases in terms of statements or other models, such as algebraic ones.		
Design	• Use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

Self-Reflection: Math and Me

The following self-reflection questionnaire serves to help you think about your relationships with mathematics. Read each sentence and rate how much you agree with each sentence, selecting from strongly disagree (SD), disagree (D), agree (A), or strongly agree (SA).

I make excellent grades on math tests				
I have always been successful with math				
Even when I study very hard, I do poorly in math				
I got good grades in math on my last report card				
I do well on math assignments				
I do well on even the most difficult math assignments				
Seeing adults do well in math pushes me to do better				
When I see how my math teacher solves a problem, I can picture myself solving the problem in the same way				
Seeing kids do better than me in math pushes me to do better				
When I see how another student solves a math problem, I can see myself solving the problem in the same way				
I imagine myself working through challenging math problems successfully				
I compete with myself in math				
My math teachers have told me that I am good at learning math				
People have told me that I have a talent for math				

	SD	D	A	SA
Adults in my family have told me what a good math student I am				
I have been praised for my ability in math				
Other students have told me that I'm good at learning math				
My classmates like to work with me in math because they think I'm good at it				
Just being in math class makes me feel stressed and nervous				
Doing math work takes all of my energy				
I start to feel stressed out as soon as I begin my math work				
My mind goes blank, and I am unable to think clearly when doing math work				
I get depressed when I think about learning math				
My whole body becomes tense when I have to do math				

Portfolio and Reflection

Today is the final opportunity for you to look back at the work that you have done, revise or improve it as necessary, and write a cover letter, which will be addressed to your new ninth grade teacher in the fall. For your portfolio, you should select at least one example from each module of work that best represents your understanding and growth over time. After you have collected samples from each of the modules, your cover letter will be a reflection.

Your cover letter should address:

- ***What connections do you see?*** As you look across the different topics (patterns, equivalence, and networks) and the different domains of mathematics (algebra, statistics, and geometry), what similarities and links do you see across subjects? How do you think these connections will help you as you study other ideas in mathematics?
- ***How have you grown mathematically?*** Looking back to where you were three weeks ago, how do you think your opinions about yourself and mathematics have changed? What practices in math are you more confident about?
- ***What helps you learn?*** Under what conditions during these modules did you feel particularly successful? What were the supports that assisted you? What should your new teacher know and offer you in the new year?
- ***What else should your teacher know about you?*** What are your other strengths and interests?

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