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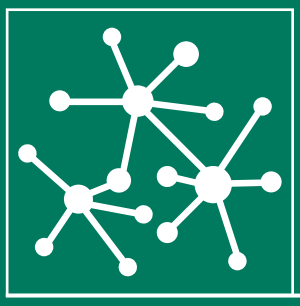


TEACHER MANUAL

Modeling Networks and Surfaces

Module Overview

In this module, students explore and apply ideas from "graph theory," which is a systematic approach to studying and quantifying characteristics of different networks. Students learn how to model relationships using representations to show the connections between key objects. They collaborate to analyze, construct, and deconstruct different representations on graphs, with attention to arguments about what is possible or not possible to do in a variety of applied problems. For English Learners, this module offers explicit supports for them to construct viable arguments and critique the reasoning of others; explore and construct key invariants; and develop applications to a wide variety of contexts.

Conceptual Understandings	Mathematical Practices	Language Practices
• Networks can be characterized by parameters across multiple dimensions. • Construction and dissection of networks is an approach to identifying invariants. • Invariants such as the Euler characteristic depend on the topology of the underlying surface.	• Reason abstractly and quantitatively (SMP2). • Attend to precision (SMP6). • Look for regularity in repeated reasoning (SMP8).	• Articulate conditions and assumptions in solving problems. • Coordinate representations and describe continuity and change across transformations. • Construct arguments for the impossibility of networks on some surfaces.

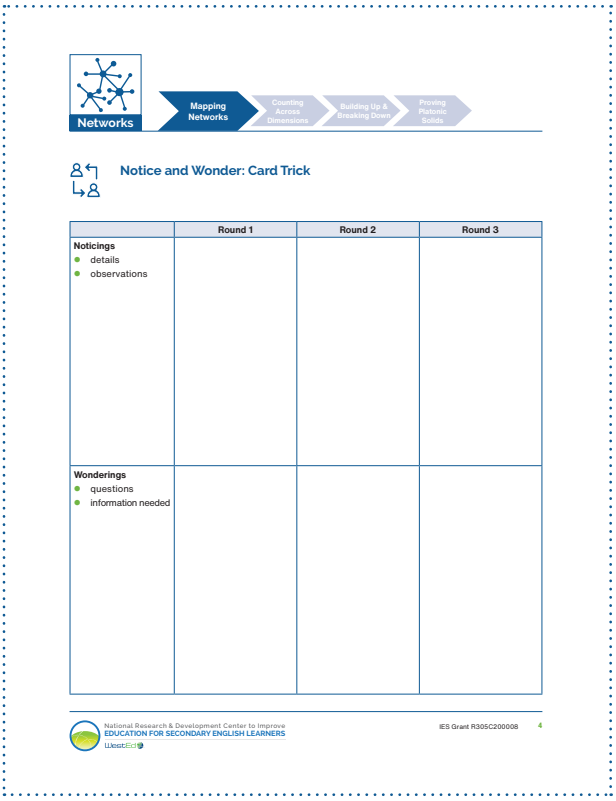
Overview of Content and Objectives

The arrow on the next page displays how students will begin by exploring relationships and how to map them in networks. The activities in this module seek to expand students' understandings of connections and relationships by presenting graph theory as the study of these networks of connections. The activities also facilitate student learning of key analytical practices, including the Standards for Mathematical Practice listed in the above table. Sequences of activities are offered to students as they extend their language use.

	Mapping Networks	Counting Across Dimensions	Building Up & Breaking Down	Proving Platonic Solids
Content Objectives: Students will:	• Understand the relationships that networks model • Solve problems given constraints	• Identify and count key elements of networks • Understand options for representation	• Calculate and compare key invariants • Generalize to nonplanar surfaces	• Identify additional features of regular graphs • Apply duality to reveal relationships
Tasks at a Glance	• Notice and Wonder: Card Trick • The Utilities Problem • Introducing Networks	• Sort and Label: Networks • Defining and Identifying Dimensions • Count and Record • Scribble, Pass, and Count • Two Problems (Venn Diagrams and the Handshake Problem)	• Step it Up • Revising the Two Problems • Considering Other Surfaces	• Tessellations in the Plane • Platonic Solids

Notice and Wonder: Card Trick

The *Notice and Wonder* task focuses student attention on the steps within a card trick and sets the stage for conversations on drawing and making connections.



Purpose and Rationale

The purpose of this activity is to surface students' ideas about connections within and across the card trick. This activity provides an engaging context for students to think about modeling with networks that show the movement of cards during the trick. The activity also serves as an occasion for students to think about how to prove the trick will always work.

Prepare

1. Secure the materials for the card trick—21 distinct playing cards. Review the steps of the card trick below and determine how you will group students when they share their noticings and wonderings.
2. Before you begin, note at least three noticing that you anticipate your students will record during the card trick:
   -
   -
   -
   -

Enact and Observe

3. Distribute the *Notice and Wonder* handout to each student. Then, enact the card trick.

Card Trick Steps

- For each round, deal out the 21 cards into 7 rows, going across from left to right. The cards, when placed upon the table, should consist of 7 rows of 3 columns of cards. Deal the cards face up so that they are visible; stacking the subsequent rows of cards will enable you to pick up the columns as stacks quickly.
- Ask students as a whole group to silently pick a secret card as you turn your back. Tell them that they should all memorize what that card is throughout the rest of the trick.
- Tell the group that you will ask not what their card is, but rather what column their card is in. When students tell you, confirm with everyone that the card is indeed in that column.
- Gather each column in order. Stack the column that was identified by students in the middle, while placing the other two columns, as stacks, on top and bottom.
- Deal out the cards again, and ask again for the column that cards are in. Repeat this for three full iterations. Then tell students that you know what their card is. Point at the card in the center row, center column and tell students that that was their secret card.

4. Prompt students to record what they notice and what they wonder, and then repeat the trick. Specifically prompt students to consider how the teacher is able to identify the card, why the trick "works." and which moves in the card trick are critical to why the trick works, specifically, that the cards move towards the middle and what card trick moves leads to the card placement.
5. Do the trick a total of three times. Have students complete their noticing and wonderings after each round of the trick, and then ask students to identify any connections across what they noticed and wondered.
6. Students will then work with a partner or small group to share their ideas. In each group, one student will share an idea, and then another student will repeat that idea and then share another idea. Students should continue until they have shared all of their ideas.
7. Take note of what students share and be prepared to select two pairs of students to share their ideas with the full group.

Close and Connect

8. Thank students for discussing what they noticed and wondered, and then ask two pairs of students to report out what they noticed and wondered. Highlight language, such as connections, drawings, mapping, that you have heard students use that will be useful in later activities.
9. Then, tell students that in the next activity they will explore three problems that involve making and drawing connections.

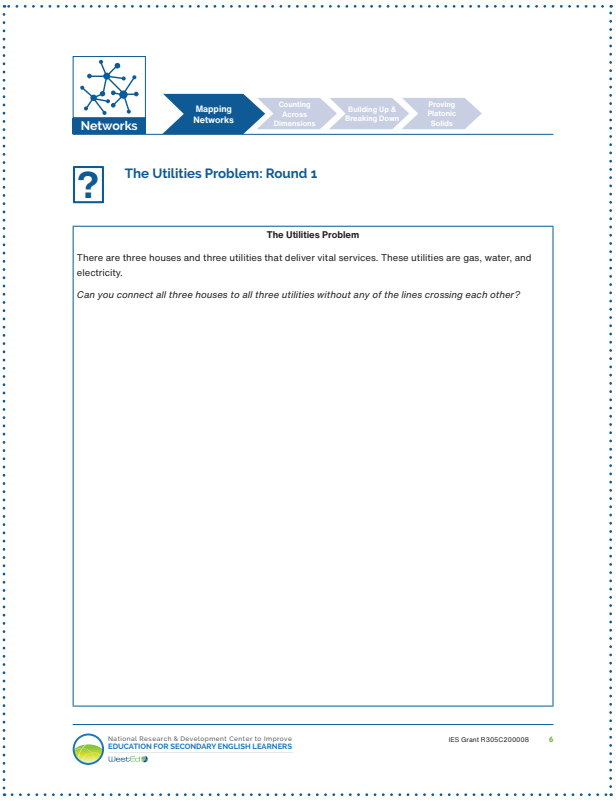
Student Work Samples on the *Card Trick Notice and Wonder*

	Round 1	Round 2	Round 3
Noticings • details • observations • connections within	I notice how she cut the deck into 3 columns and 7 in each row 	I notice how you continue to shuffle the cards once a round is done.	I notice how you continued to put the row the card was in your palm, first
Wonderings • questions • information needed • connections beyond	I wonder why she cut the deck into 3 columns by 7	I wonder how you figure out the card/ I wonder if there is a trick.	I wonder how you kept guessing the card

	Round 1	Round 2	Round 3
Noticings • details • observations • connections within	• Always puts the selected column in the middle of the others • She never reshuffled	• It was always somewhere in the middle of the 2nd column never at the top or bottom	•
Wonderings • questions • information needed • connections beyond	• Why she did that (19)? • How she kept track • why she didn't reshuffle	• How she kept track? • How she knew?	• How she guessed

The Utilities Problem

In this activity, students explore a simply framed problem ask challenges them to consider particular conditions and the embedded assumptions.



Purpose and Rationale

The purpose of this task is for students to identify the assumptions they are making and the conditions under which networks which would be possible, specifically noting any assumptions or conditions that are similar across tasks. The utilities problem is set in a context where student can generate multiple intuitive ideas about surfaces and conditions for building networks.

Prepare

1. Before you begin, the utilities problem and anticipate what are potential student stopping points. That is, where will students end up before they feel "stuck" with the problem.
2. Review the student work samples at the end of this task to identify some of these trends.
3. Students will work in partners on this activity.

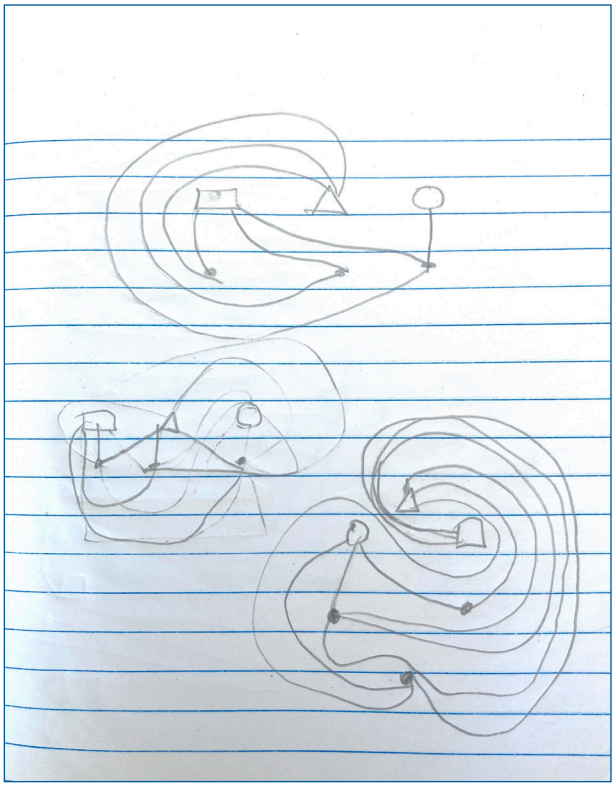
Enact and Observe

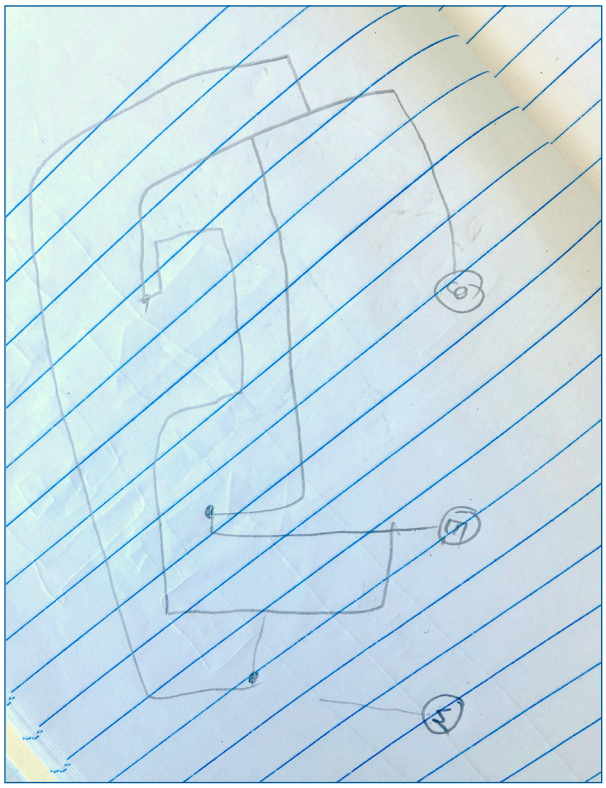
4. Distribute the *Utilities Problem* handout. Prompt students to read the instructions and work with a partner to try to solve the problem. When students become stuck, offer the first modification, which is to pretend that the surface is more like the surface of the earth. Have students explicitly state how they are unwrapping a spherical surface into a flat piece of paper. That is, have them talk about where lines connect off the boundary of the paper.
5. When students then get stuck on the surface of the earth, ask them if there are other assumptions that they could make that would enable them to solve the problem. Have students explain how those assumptions would correspond to another surface on which the network would be built.

Close and Connect

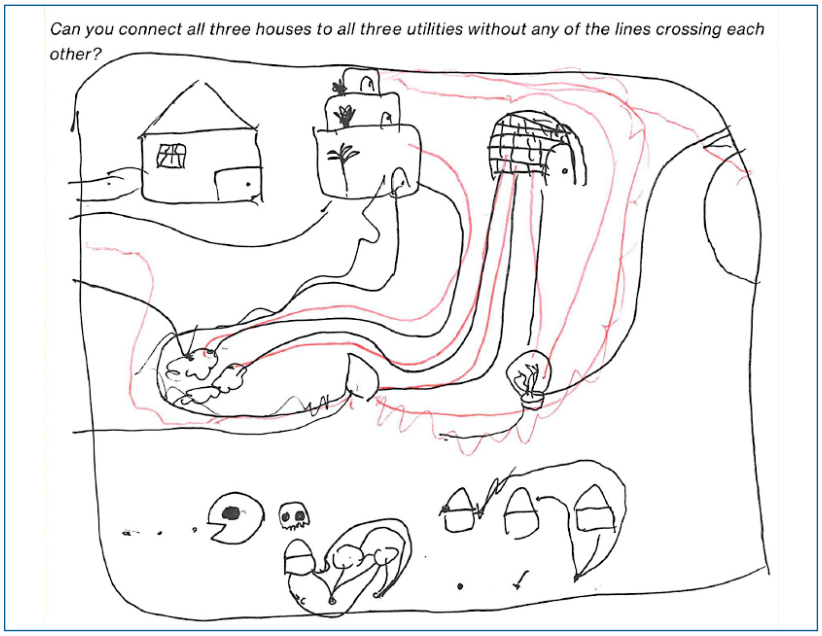
6. Close the activity by asking selected students to share the conditions they could try for each problem to make it possible.
7. Then, tell students that they will next read more about networks and consider how networks can map social relationships.

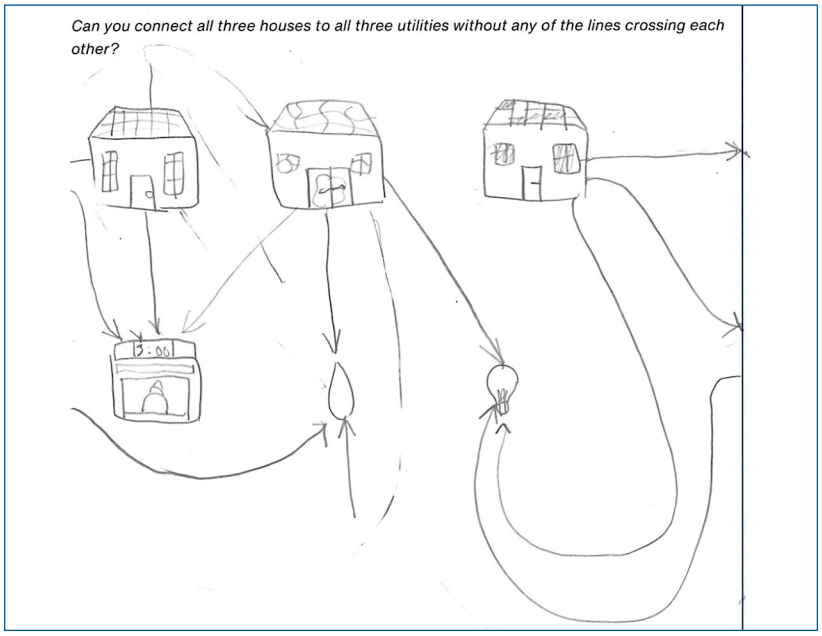
Student Work Samples on *Utilities Problem: Round 1*





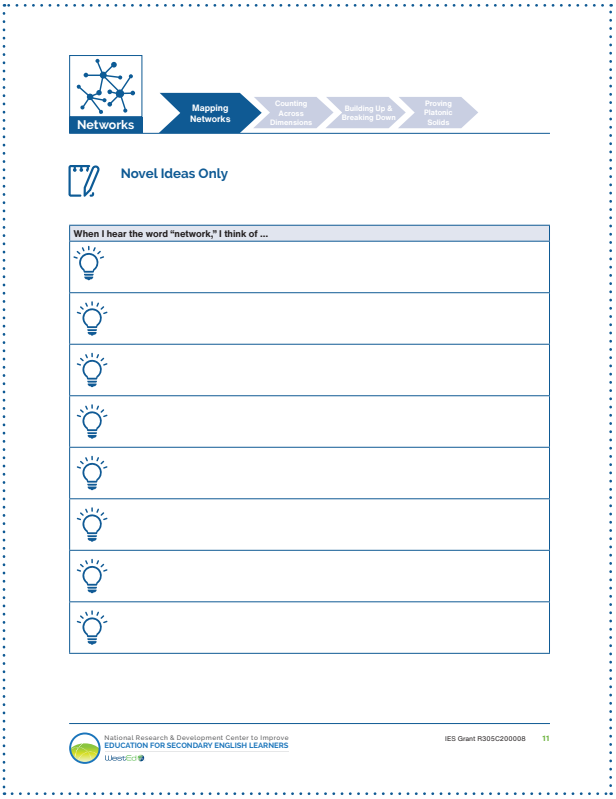
Student Work Samples on *Utilities Problem: Round 3*





Novel Ideas Only

The *Novel Ideas Only* task elicits what students already know about networks and emphasizes the valuable prior knowledge that they bring to this unit.



Purpose and Rationale

The purpose of this activity is to surface students' prior knowledge and experiences.

This activity provides an opportunity for students and the teacher to hear students' current ideas about networks and to connect to students' ideas of networks before inviting them to expand their skills and knowledge in subsequent activities. The small group structure provides an opportunity for students to share their ideas with a small group of peers; then, in the full group, students hone their listening skills as other groups present. As you enact this activity, be ready to listen carefully for student ideas that you may want to highlight and encourage during later activities. Connecting later learnings to students' current ideas can help students make connections and build a robust understanding of networks.

Prepare

1. Before you begin, note at least three ideas that you anticipate your students use to complete the prompt, "When I hear the word "network," I think of...". Note your ideas here:
   -
   -
   -
   -
2. Review both parts of this activity and determine how you will group students. Students will be working together for about 10 minutes.

Enact (what to do) and Observe (what to look/listen for)

3. Distribute the *Novel Ideas Only* handout to each student. Read the directions for small group work aloud.
4. Give students 3 minutes for small group work. In each group, one student will share an idea in response to the prompt, another student will repeat it, and all students will write it down. Students continue sharing, repeating, and writing ideas for 3 minutes.
5. Note that everyone in the small group should have the exact same information on their papers.
6. After students work in groups for 3 minutes, lead a full-class sharing where students report out their "novel ideas:" Ask one student from one group to stand and read all the ideas that their group collected. Other students should listen carefully and fill in the lightbulb if one of their ideas is read aloud. Then, the next group should read aloud only the ideas from their group that were not yet shared. Continue this process until all groups have shared. To assist in keeping track of what groups have shared, ask groups to sit down after they have shared their novel ideas.
7. After students have completed reporting, encourage them to keep listening, and to add to their list if they hear a new idea that they would like to include.
8. Listen for one or two repeated ideas shared by students or a theme across student reporting that you can use during the lesson closing.

During small group work:

- Encourage students to listen carefully to their classmates' ideas.
- Prompt students to take turns sharing and repeating ideas.
- Pause the activity after 3 minutes, as this is enough time for students to write their initial ideas.

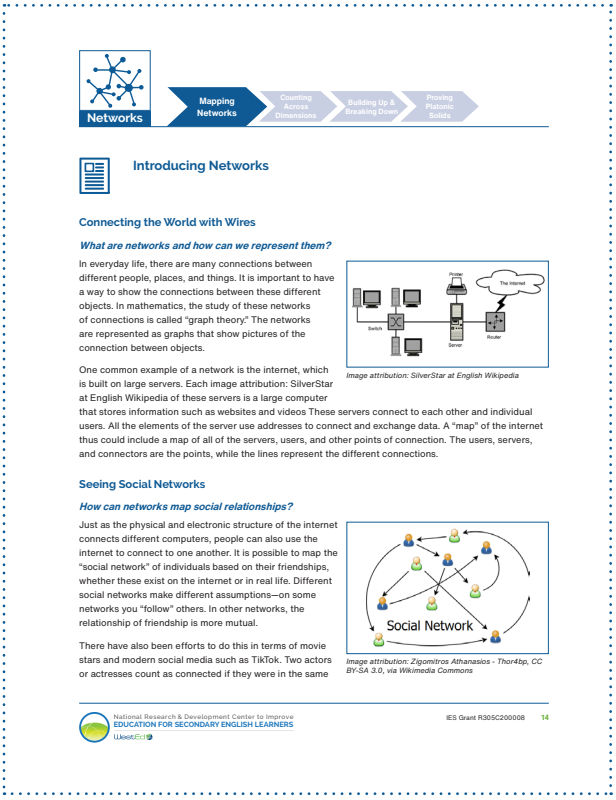
Close and Connect

9. Close the activity by thanking students for reporting out their novel ideas and then sharing a theme that you heard across student responses. Highlight language that you have heard students use that will be useful in later activities.

Remember that the goal of the activity to collect students' ideas and encourage their thinking; the goal is not to write out an exhaustive list of all ideas about networks. Refrain from asking students to write all ideas they hear.

Introducing Networks

In this activity, students will read about networks, how to represent them and how to use them to map relationships.


Networks
Mapping Networks
Counting Across Dimensions
Building Up & Breaking Down
Proving Platonic Solids
Introducing Networks
Connecting the World with Wires
What are networks and how can we represent them?
In everyday life, there are many connections between different people, places, and things. It is important to have a way to show the connections between these different objects. In mathematics, the study of these networks of connections is called "graph theory." The networks are represented as graphs that show pictures of the connection between objects.
Printer
The Internet
Switch
Server
Router
Image attribution: SilverStar at English Wikipedia
One common example of a network is the internet, which is built on large servers. Each image attribution: SilverStar at English Wikipedia of these servers is a large computer that stores information such as websites and videos These servers connect to each other and individual users. All the elements of the server use addresses to connect and exchange data. A "map" of the internet thus could include a map of all of the servers, users, and other points of connection. The users, servers, and connectors are the points, while the lines represent the different connections.
Seeing Social Networks
How can networks map social relationships?
Just as the physical and electronic structure of the internet connects different computers, people can also use the internet to connect to one another. It is possible to map the "social network" of individuals based on their friendships, whether these exist on the internet or in real life. Different social networks make different assumptions—on some networks you "follow" others. In other networks, the relationship of friendship is more mutual.
Social Network
Image attribution: Zigomitros Athanasios - Thor4bp, CC BY-SA 3.0, via Wikimedia Commons
There have also been efforts to do this in terms of movie stars and modern social media such as TikTok. Two actors or actresses count as connected if they were in the same
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Purpose and Rationale

The purpose of this reading is to provide students with some additional information on networks and an opportunity to clarify their thinking on these ideas while reading with a partner. The reading and the *Clarifying Bookmark III* can encourage students to deepen their understanding of the content while also highlighting any points that need to be clarified. Through taking turns and working with a partner, students practice discussing mathematical texts using the *Clarifying Bookmark III.*

Prepare

1. Read the texts on *Connecting the World with Wires* and *Seeing Social Networks.* Consider questions that students may have.
2. Draw at least one of your social networks to prepare to discuss drawing social networks with students. Consider the center, diameter, and radius. Make your drawing here so you can be ready to share with students.

Enact and Observe

3. Organize students into groups of two and distribute a copy of the texts on *Connecting the World with Wires* and *Seeing Social Networks* and the *Clarifying Bookmark III*. Review the directions. Tell students that they will each take a turn reading a section of the text. After the first student reads, the student will share reflections or questions; students can choose a strategy from the *Clarifying Bookmark III* and use one of the formulaic expressions to share ideas. The second student will also share ideas and choose a formulaic expression from the *Clarifying Bookmark III*, if interested in using it. Students will alternate reading and continue taking turns until they have finished reading.

4. Students will then draw their own social networks. Encourage students to consider the center, diameter, and radius of their social networks and how to represent them. Share your own social networks with students if an example is helpful for their conversations.

Close and Connect

5. Close this activity by asking some students to share with their partner one new idea that they learned. Encourage students to share with the whole class what their partner said.
6. Then, ask two students to share what they used as center, diameter, and radius of their social networks and why.
7. Prompt students to complete the *Day 1: Daily Writing Prompt* about the activities they completed today, including something they learned today and a question they still have.
8. Then, tell students that on Day 2 they will learn more about networks and, in particular, consider how to count the dimensions of them.

Reflect After Day 1

After the last activity of *Day 1: Mapping Networks* has concluded, please reflect on student learning by responding to the following questions:

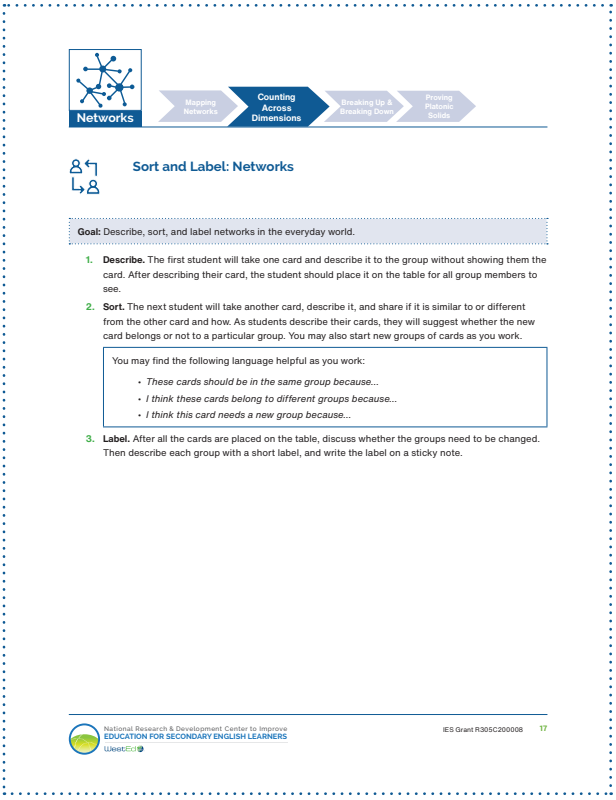
1. What assumptions that students made related to *The Utilities Problems* were unexpected to you?

2. For which students did the activity of drawing their social networks work well? How do you know? In what ways did students use the ideas of center, diameter, and radius in their social networks?

3. What did you notice about students' discussions today? What questions did they have as they were reading and how did they discuss them with their partner?

Sort and Label: Networks

There are different types of connections that can be made with vertices, edges, and faces. As students identify these dimensions in networks, they can see how to define these networks and use the data to classify them. In this activity, students describe networks and then work together to sort them, identifying dimensions that define them.


Networks
Mapping Networks | Counting Across Dimensions | Breaking Up & Breaking Down | Proving Platonic Solids

Sort and Label: Networks

Goal: Describe, sort, and label networks in the everyday world.

1. **Describe.** The first student will take one card and describe it to the group without showing them the card. After describing their card, the student should place it on the table for all group members to see.
2. **Sort.** The next student will take another card, describe it, and share if it is similar to or different from the other card and how. As students describe their cards, they will suggest whether the new card belongs or not to a particular group. You may also start new groups of cards as you work.

You may find the following language helpful as you work:

- *These cards should be in the same group because...*
- *I think these cards belong to different groups because...*
- *I think this card needs a new group because...*

3. **Label.** After all the cards are placed on the table, discuss whether the groups need to be changed. Then describe each group with a short label, and write the label on a sticky note.

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Purpose and Rationale

The purpose of this task is for students to identify common features and key dimensions of networks. The *Sort and Label* activity allows students to practice using the characteristics of networks that they have identified to sort and explain networks.

Prepare

1. Review the cards and identify different ways that students may sort the networks.
2. Consider how to group students and how different grouping may facilitate student learning opportunities.

Enact and Observe

3. Organize students into groups of four. Distribute the *Sort and Label* handout.
4. Read the directions for the task aloud. Tell students that they will sort the cards one at a time, discussing the order as they work. On each turn, a student will take a card and describe it to the group without showing them the card. After describing their card, the student should place it on the table for all group members to see. In subsequent turns, students will describe their cards and suggest whether the new card belongs in relation to the other cards.

5. While students are working, make note of how they are sorting the cards, and prompt student conversations using the sample formulaic expressions. If students ask you for help, encourage students to note the similarities and differences across cards.
6. After all the cards are placed on the table, students can discuss whether the groups need to be changed.

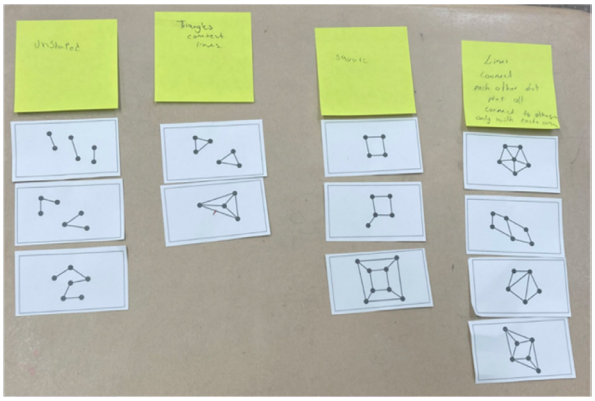
Close and Connect

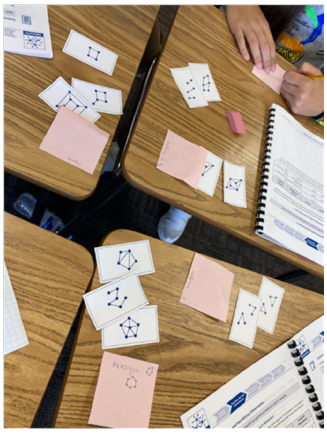
7. Close the activity by sharing two samples of how cards were sorted and noting how students could use different dimensions to sort the cards. Tell students that in the next activity they will learn more about the dimensions in networks and how to count them.

Share these formulaic expressions for students to use during small group work:

- *This card should be here because...*
- *I think this network is similar to or different from... because...*
- *This card needs to be moved because...*

Sample Student Work and Sample Labels

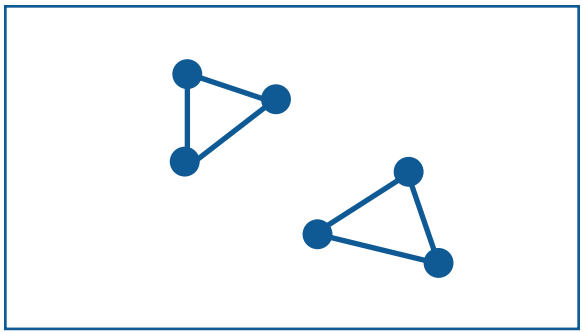


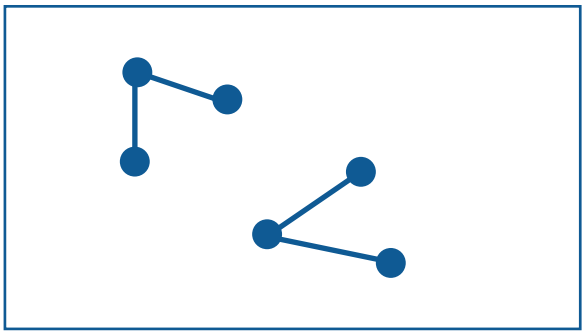


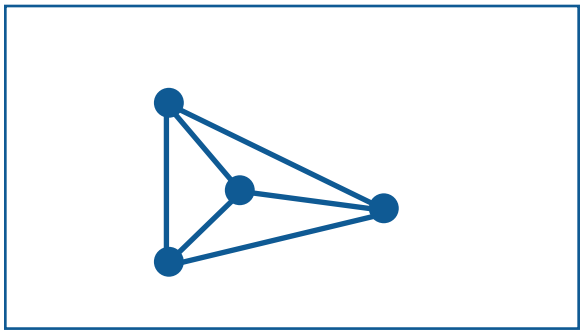
Group	Labels				
1	Unshaped	Triangles and connected lines	Squares	Lines connect to each other but not all connect to others	
2	Hexagons	Square	Triangles	¾ of a triangle	
3	No faces	One face	Two faces	Three faces	Five faces

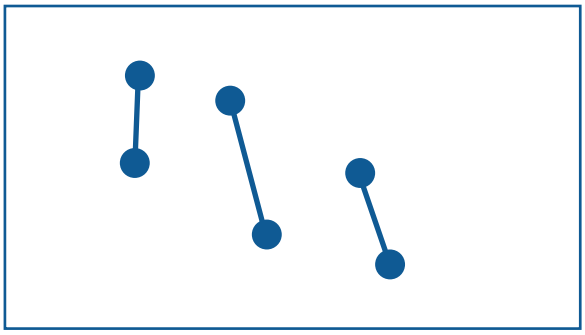
Table 1. Student-Generated Categories for Networks

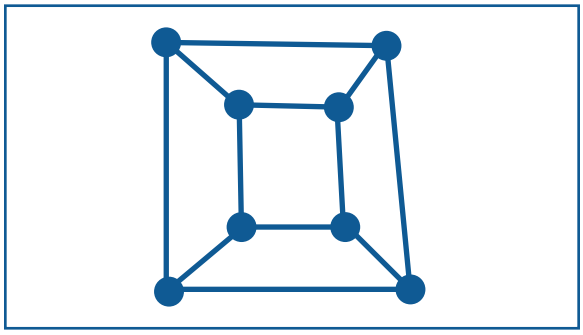
Sort and Label: Network Cards

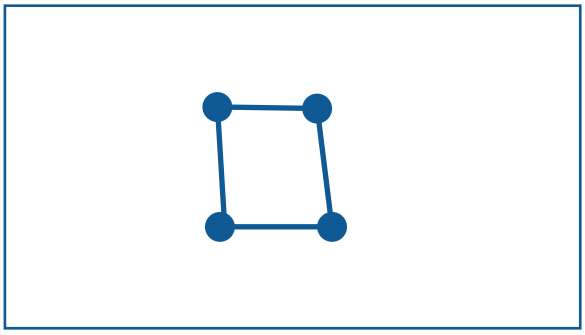


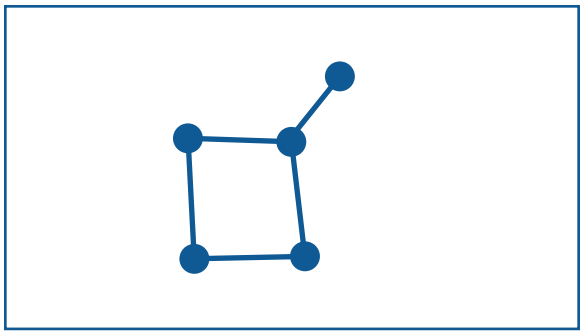


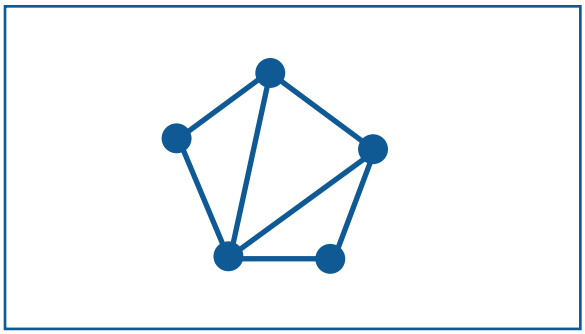


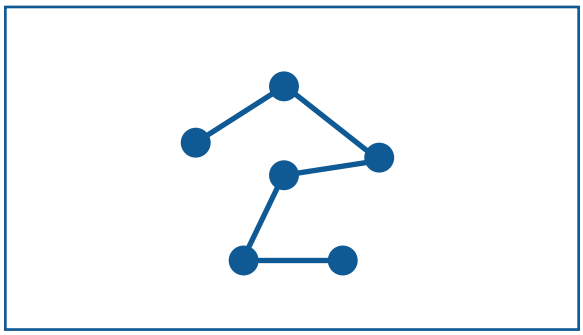


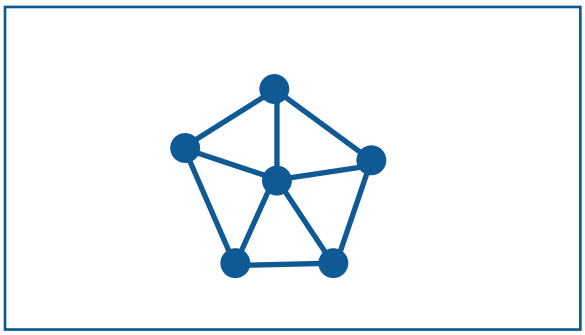


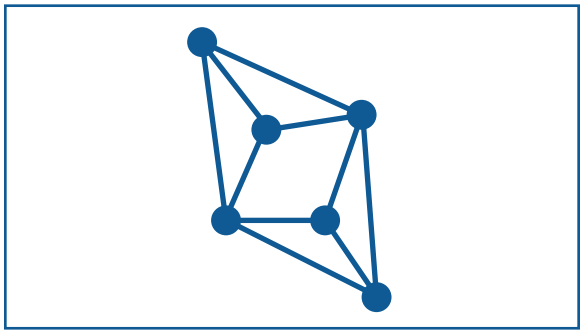


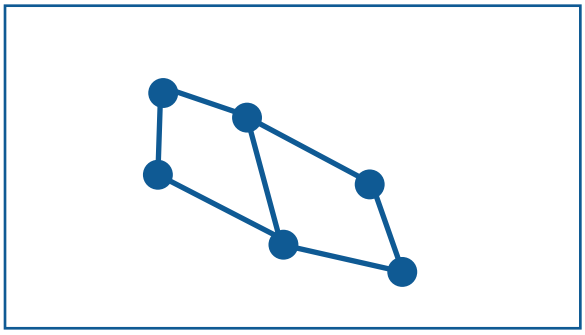






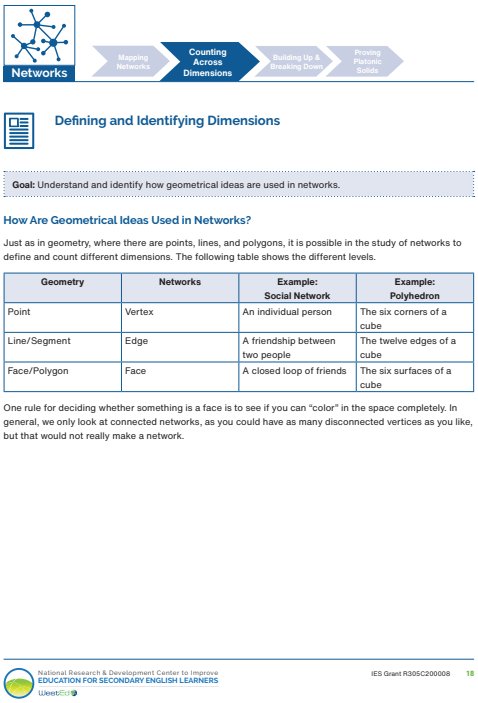






Defining and Identifying Dimensions

In this activity, students will read about how geometric ideas are used in networks and apply these understandings to networks from the *Sort and Label* activity.


Networks
Mapping Networks | Counting Across Dimensions | Building Up & Breaking Down | Proving Platonic Solids

Defining and Identifying Dimensions

Goal: Understand and identify how geometrical ideas are used in networks.

How Are Geometrical Ideas Used in Networks?

Just as in geometry, where there are points, lines, and polygons, it is possible in the study of networks to define and count different dimensions. The following table shows the different levels.

Geometry	Networks	Example: Social Network	Example: Polyhedron
Point	Vertex	An individual person	The six corners of a cube
Line/Segment	Edge	A friendship between two people	The twelve edges of a cube
Face/Polygon	Face	A closed loop of friends	The six surfaces of a cube

One rule for deciding whether something is a face is to see if you can "color" in the space completely. In general, we only look at connected networks, as you could have as many disconnected vertices as you like, but that would not really make a network.

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Purpose and Rationale

The purpose of this activity is to provide students with some information on where geometric ideas are in networks. Students will have an opportunity to apply these ideas to networks that they first explored in the *Sort and Label* activity. This activity builds on students' current understandings by providing key terms and ideas to networks and prepares them for applying what they learn about counting dimensions to later tasks.

Prepare

1. Read the text on *How Are Geometrical Ideas Used in Networks*? and be prepared to describe what a connected network looks like.
2. Complete the table *Count and Record*, which uses the networks from the Sort and Label activity.

Enact and Observe

3. Organize students into groups of two and distribute the *Defining and Identifying Dimensions* handout.
4. Review the directions. Tell students that they will each take a turn reading a section of the text with a partner and then discuss what they learned about how geometrical ideas are used in networks. After students finish reading, they will complete the table, *Count and Record*, which uses the networks that they explored in the *Sort and Label* activity.

During small group work, encourage students to share questions with each other that they have from the reading. Students can use ideas from *Clarifying Bookmark III* or from other bookmarks as needed.

5. Review students' responses in the table as they are working and point them to the text if they need to clarify what vertices, edges, and faces are and how they are used. Prompt students to share what they notice about the networks and then number of each dimension. See the sample student work:
6. Students will then read the text *How Do Regular Shapes Look in Three Dimensions*? As students are reading, prompt them to discuss any questions they have.

Close and Connect

7. Close this activity by asking some students to share any new ideas from the reading.
8. Then, ask students to share any patterns that they noticed as they were counting the dimensions. When students are sharing, prompt them to be specific, sharing the network, vertices, edges, and faces.
9. Review students' responses in the table as they are working and point them to the text if they need to clarify what vertices, edges, and faces are and how they are used. Prompt students to share what they notice about the networks and then number of each dimension.

Student Work Sample on *Count and Record*

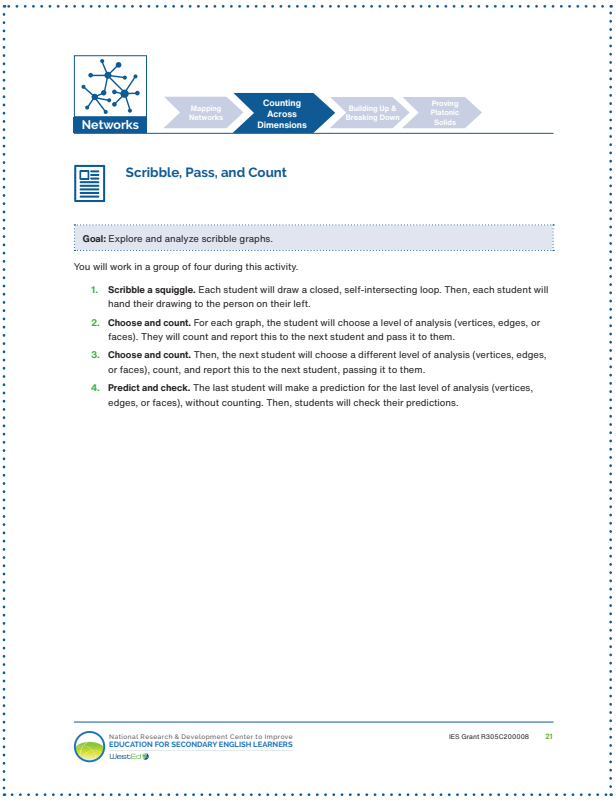
Count and Record

Review the networks below and identify the number of vertices, edges, and faces.

Network	Number of Vertices	Number of Edges	Number of Faces
	4	4	1
	5	5	1
	8	12	5
	6	7	2
	4	6	3
	6	10	5
	6	5	0
	5	7	3
	6	10	5

Scribble, Pass, and Count

In this activity, students will create a scribble graph and both count and predict the vertices, edges, and faces.



Purpose and Rationale

The purpose of this activity is to provide students with an opportunity to review and test levels of analysis, while exploring their own scribble graphs. This activity builds on students' developing understandings of different levels of analysis and prepares them for applying what they learn about counting dimensions to later tasks.

Prepare

1. Read the instructions for *Scribble, Pass, and Count.*

Enact and Observe

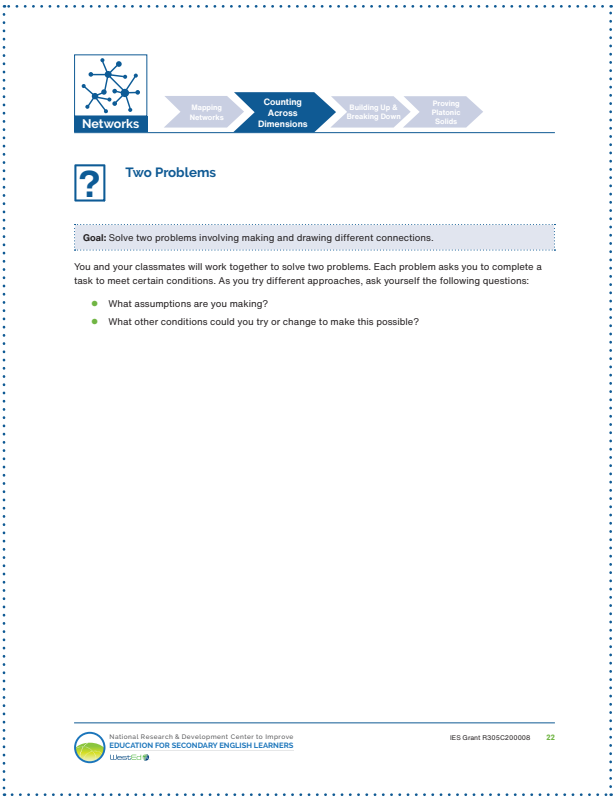
2. Organize students into groups of four. Review the instructions. Ask students to begin the activity with their own scribble graph, and then pass their paper to the left. When receiving the paper, the next student will choose a level of analysis (vertices, edges, or faces). They will count and report this to next student and pass it to them.
3. The last student will make a prediction for the last level of analysis (vertices, edges, or faces), without counting. Then, students will check their predictions.

Close and Connect

4. Close this activity by asking some students to share any new ideas from the reading.
5. Then, ask students to share any patterns that they noticed as they were counting the dimensions. When students are sharing, prompt them to be specific, sharing the network, vertices, edges, and faces.

Two Problems

In this activity, students explore two problems that are "complete" networks based upon two familiar contexts: Venn diagrams and drawings of handshakes.



Purpose and Rationale

The purpose of this task is for students to engage in problem solving and to begin to identify some of the characteristics of netoworks that satisfy certain characteristics. Venn diagrams are familiar representations used to representations to either perform counting or to compare and contrast attributes. Handshakes were a context in the first module and now the problem is about drawing them without additional crossings of a flat surface.

Prepare

1. Before you begin, try both problems and anticipate what are potential student stopping points. That is, where will students end up before they feel "stuck" with the problem.
2. Review the student work samples at the end of this task to identify some of these trends.
3. Students will work in partners on this activity.

Enact and Observe

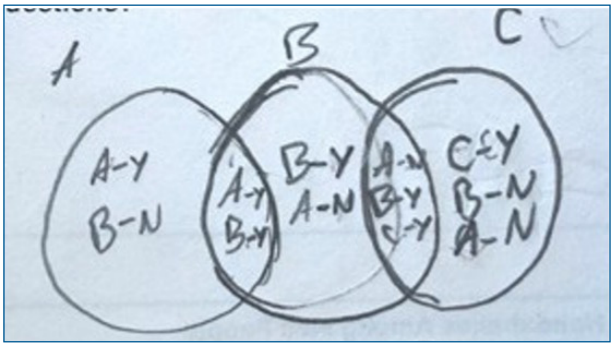
4. Prompt students to read the instructions on the *Two Problems* handout and work with a partner to try to solve each problem.
5. Encourage students to notice and to count what they see in the picture. They do not need to have this complete, however, as it will be defined more clearly in the following task.

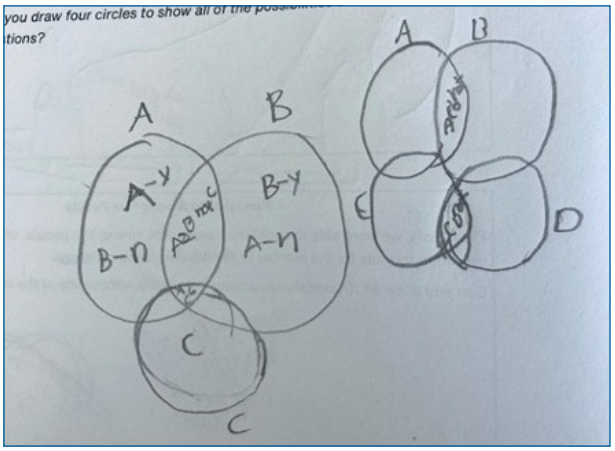
Close and Connect

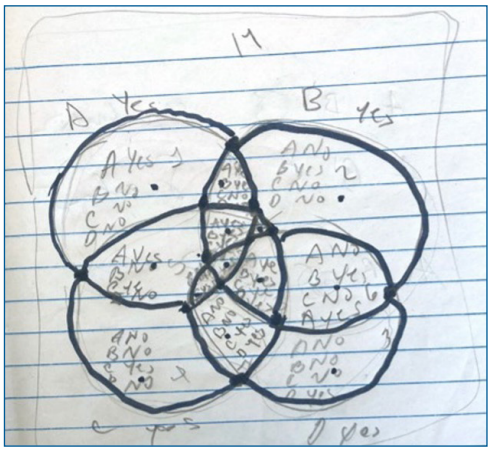
6. Close the activity by asking selected students to share the conditions they could try for each problem to make it possible.
7. Ask students to complete the *Day 2: Daily Writing Prompt* about the activities they completed today, including something they learned today and a question they still have.
8. Then, tell students that on Day 3 they will continue to learn more about networks and their dimensions and revisit the problems using what they learn about networks.

Student Work Samples on Venn Diagram with Four Sets

Students attempted to draw all possible intersections of four sets.







Reflect After Day 2

After the last activity of *Day 2: Counting Across Dimensions* has concluded, please reflect on student learning by responding to the following questions:

1. What did you notice about how students shared, sorted, and labeled the networks in the *Sort and Label* activity? In what ways did they reason using geometric ideas?

2. For which students did the *Sort and Label* activity foster student mathematical discourse? In what ways, if any, has student talk during the *Sort and Label* activities changed since the first *Sort and Label* activity in Module 1?

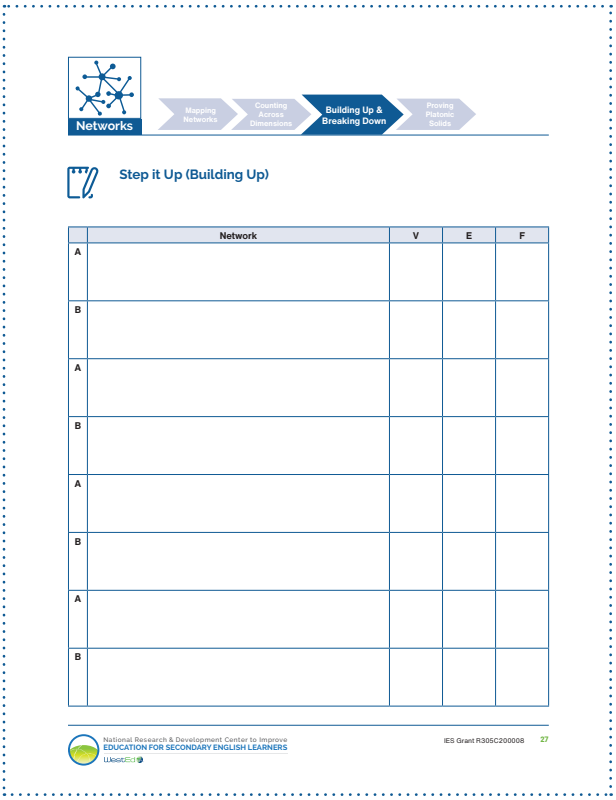
3. What ideas about or patterns within networks did students share after the *Defining Dimensions* activity, reading, and related table?

Step it Up

Students co-create and draw networks, adding vertices, edges, and faces to create new networks.

Purpose and Rationale

In the *Step it Up* task, students co-construct increasingly complex networks and as they track the elements across different dimensions. By creating networks themselves, students can explore the process as well as the final product and can investigate what changes and does not change as networks become more complex.



Prepare

1. Determine how you will group students for this activity. Consider if students should work with a partner with whom they have not worked yet.
2. Review the activity and take note of how students will create networks. For example, on which networks can students add faces? What will have to be in place first? When will students be able to add edge?

Enact and Observe

3. Distribute the *Step It Up* handout. Read the directions aloud to all students. Then, with one student, model the first two turns of creating a network and adding components. Ask students to follow along but not to copy down this sample.
4. As students begin to work with a partner, encourage them to share out loud how they are deciding what dimensions to add and any constraints on what their next steps can be.

5. When they have completed the chart, students should then discuss with their partner any constraints or conditions they noticed when creating networks and any relationships across the set of networks. Encourage students to be ready to justify their ideas.
6. Review the networks that students created and prepare to select a pair of students (or more than one pair of students) to share their ideas.

Close and Connect

7. Prompt students to share the networks that they created and any conditions or constraints they noticed.
8. Tell students that in the next task they will reconsider the problems from Day 1 and seek to apply what they have learned about networks.

Pairs discussion when creating graphs on *Step It Up* activity

One pair jointly constructed the graph shown, while engaging in the following discussion.

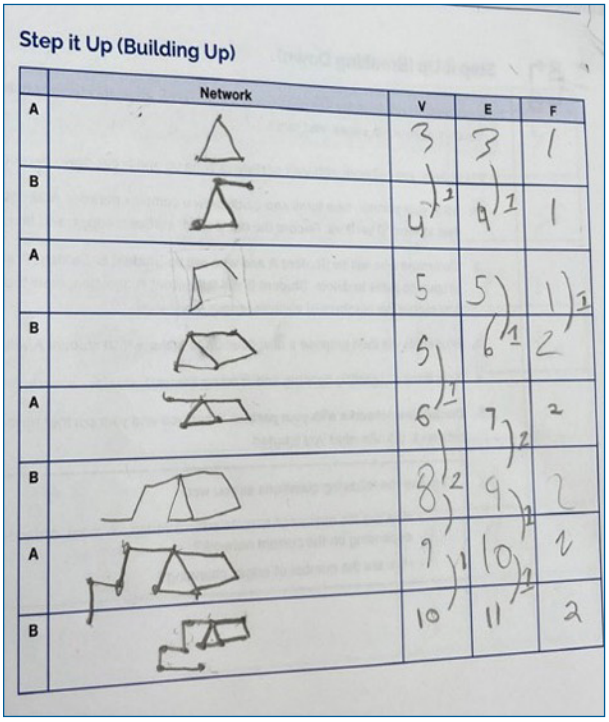
Student A: Let's start with a triangle. So we have 3-3-1.

Student B: Okay, I'm just going to add an extra point and connect it up. So there's one more vertex, and one more edge. [Writes 4, 4, 1 in the table]

Student A: I'm going to do the same thing, so now we have this long line. [Writes new counts]

Student B: Okay, so let's connect up that end to the triangle. So, now we have an extra face. [Records 5, 6, 2 in the table]

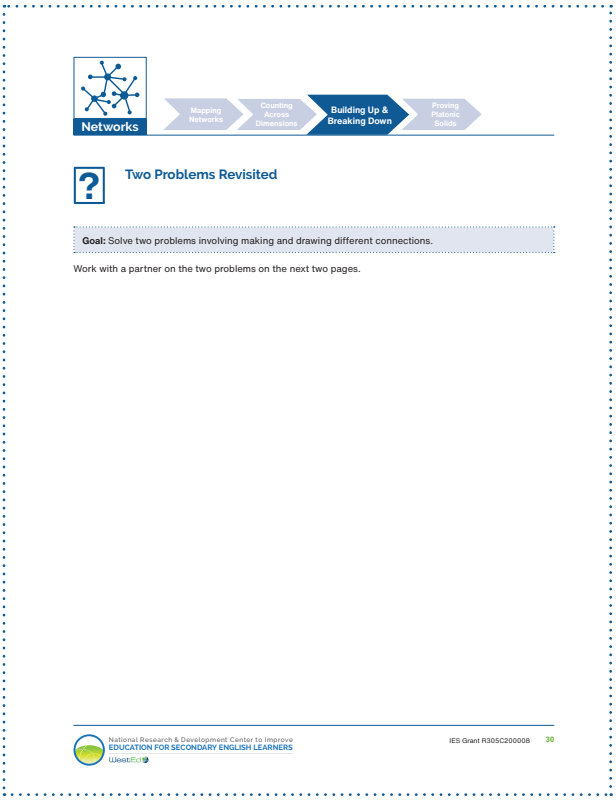
Student A: Alright, I'm going to add another line on the left, so we are up to 6 points, 7 lines, and 2 shapes inside. [Records]



Student B: Let's see what happens if I add two more points and lines. That takes us up to 8 vertices, 9 edges, and 2 faces.

Revisiting the Two Problems

Now that students have explored networks and their dimensions, they will revisit the problems shared on Day 2 and consider if those original problems could be completed.


Networks
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Two Problems Revisited

Goal: Solve two problems involving making and drawing different connections.

Work with a partner on the two problems on the next two pages.

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Purpose and Rationale

The purpose of this activity is for students to apply what they've learned about networks to Venn diagrams and handshakes. This activity supports students in making connections from what they have learned to problems that they have already started considering and applying their learning about networks.

Prepare

1. Review each of the *Two Problems* and identify what the values of vertices, edges, and faces in each would be if you could successfully complete those tasks. Complete the table with this information.
2. Clearly distinguish between counting what is there when a network is successfully drawn, versus what there should be if the network satisfies the original requirements. Review the sample student work that shows how students will tend to count what is, rather than what is the handout—which is what is for simpler cases, and then predictions for the more complex cases of (five) handshakes and (four) sets in the Venn diagram.
3. Consider grouping students with the students that they worked with on Day 1 when working on the *Two Problems.*

Enact and Observe

4. Distribute the *Two Problems Revisited* handout. Review the directions. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered. Make sure that students complete the table with what the values of

vertices, edges, and faces in each would be if you could successfully complete those tasks (not the values that they count in the task as written).

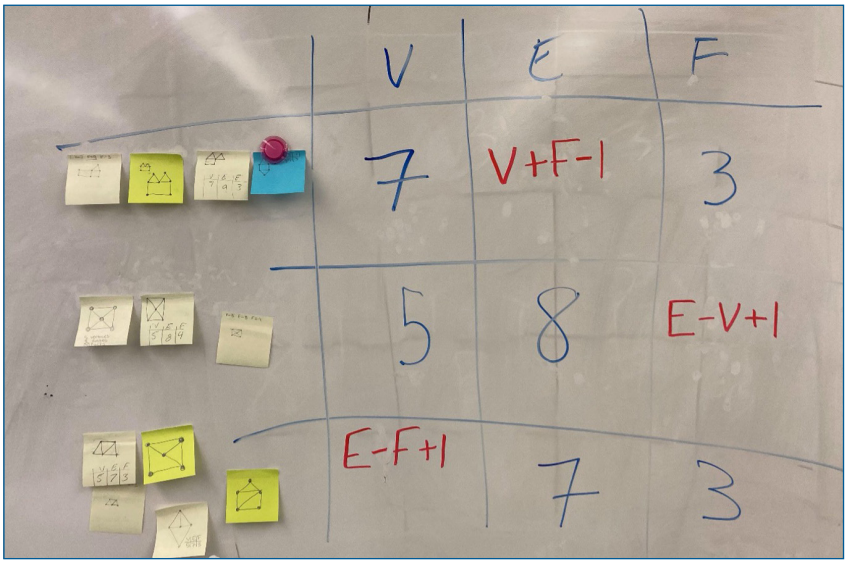
5. Prompt students to complete the *Compare and Contrast* the *Two Problems* handout and the *Part 1: Data on the Two Problems*. Students should consider the patterns that they have noticed in their construction and deconstruction tasks. Ask students to consider sharing their patterns with the full group.
6. Students should read the text on the *Part 2: Euler Characteristic* and consider how V + F – E works in each problem above.

Close and Connect

7. Ask students to share their patterns that they noticed in their construction and deconstruction tasks. Prompt students for questions about the Euler characteristics and how V + F – E applies.
8. Offer students a challenge where two of the values are given, such as V = 7 and F = 3. (See sample student work below). Ask what conclusions they can draw about the other value, in this case the number of edges.
9. Then, tell students that they will continue to consider the construction and deconstruction of networks, and in the next activity, they will consider other surfaces.

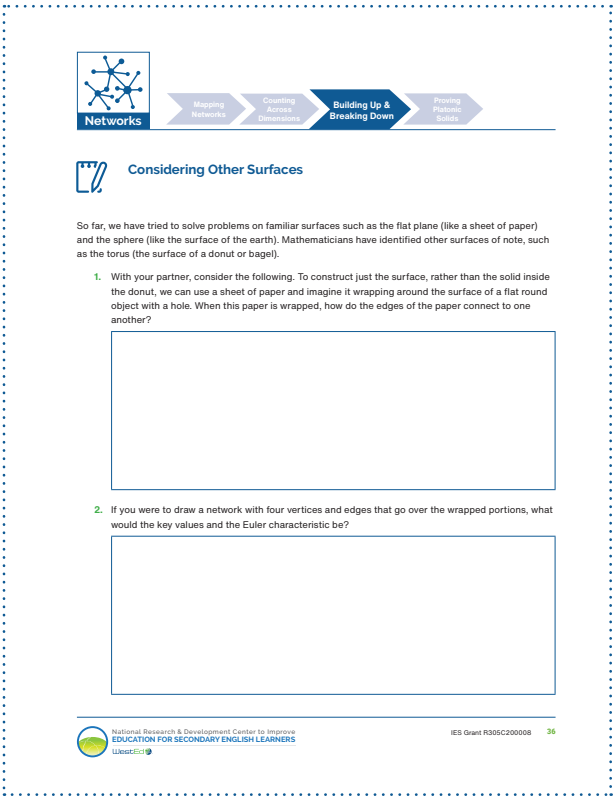
Student Work Samples

Students generated examples of graphs and then wrote examples of formulas.



Considering Other Surfaces

In this activity, students work on additional problems that consider different surfaces.



Purpose and Rationale

The purpose of this activity is for students to consider how to solve problems on other surfaces, such as the torus. This activity encourages students to apply what they have learned to new contexts and surfaces, and consider how the dimensions of vertices, edges, or faces apply (or not).

Prepare

1. Review and take notes on each of the problems in the *Considering Other Surfaces* activity.
2. Have paper available for students to use to construct another surface, as needed.

Enact and Observe

3. Distribute the *Considering Other Surfaces* handout. Prompt students to read the first question and consider how wrapping paper creates a surface.
4. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered.
5. When students work on the second prompt, encourage them to sketch a network with four vertices and edges that go over a flat round object with a hole. Ask students for the values of the vertices, edges, and faces and what the Euler characteristic would be. Encourage students to take notes so they are prepared to share their ideas with the full group.

Close and Connect

6. Ask students to share their questions that they have about constructing other surfaces. Encourage students to respond to their peers' questions by sharing what they created and what they learned from these activities.
7. Close with a discussion about what students know about the values and Euler characteristics of a torus. Prompt students to complete the *Day 3: Daily Writing Prompt* about the activities they completed today, including some explanations for why the Euler characteristic is an invariant.

Reflect After Day 3

After the last activity of *Day 3: Building Up and Breaking Down* has concluded, please reflect on student learning by responding to the following questions:

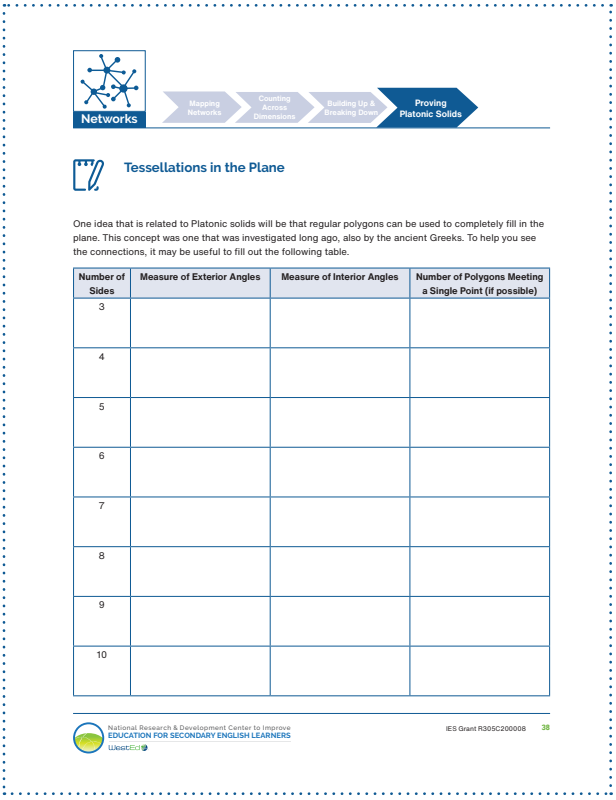
1. What surprised you about what students noticed about the constraints and conditions when creating networks in the *Step it Up* activity?

2. For which students did the *Revisiting the Three Problems* activity support students to make connections to their new learnings? How do you know?

Tessellations in the Plane

Purpose and Rationale

The purpose of this activity is for students to explore some relationships between regular "flat" graphs in the plane. This activity supports students to identify key characteristics of polygonal tilings of the plane that they will use.


Networks
Mapping Networks
Counting Across Dimensions
Building Up & Breaking Down
Proving Platonic Solids

Tessellations in the Plane

One idea that is related to Platonic solids will be that regular polygons can be used to completely fill in the plane. This concept was one that was investigated long ago, also by the ancient Greeks. To help you see the connections, it may be useful to fill out the following table.

Number of Sides	Measure of Exterior Angles	Measure of Interior Angles	Number of Polygons Meeting a Single Point (if possible)
3			
4			
5			
6			
7			
8			
9			
10			

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Prepare

1. Review and solve the table provided in the student materials. Anticipate student approaches for finding the interior and exterior angles, and your responses.
2. Decide how to group students. At this point, students will have developed significant autonomy and may choose to work independently or with partner of their own selection.

Enact and Observe

3. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered.
4. Look at how students are using tools, such a protractors or diagrams, to represent their thinking and reasoning.

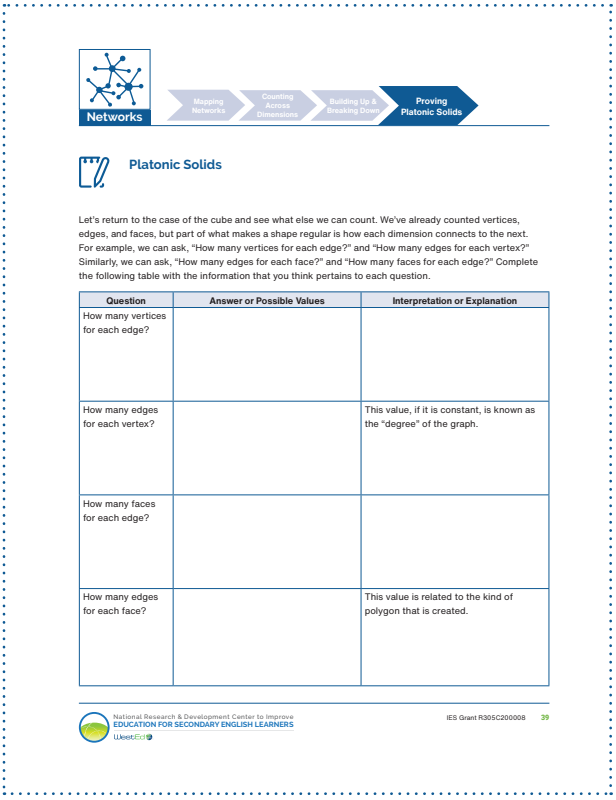
Close and Connect

5. Ask students to share their findings and arguments for why there are only three tilings possible.
6. Then, close with a discussion of what ideas students think will be useful as they extend to three dimensions and look for Platonic solids.

Platonic Solids

Purpose and Rationale

The purpose of this activity is for students to construct arguments for why certain Platonic solids are possible, but no others are. This activity helps students to organize data and see patterns and relationships across solids.



Prepare

1. Review and complete the tables in the student materials. Anticipate where students may be confused or need more guidance, such as questions about degree or other ideas. Consider what models to offer students, either in the form of nets that they can assemble or in the form of polyhedral dice or models.
2. Consider how to group students. By now, students have developed substantial autonomy and confidence, so give them some leeway in terms of how they want to work, including individually or with self-selected partners.

Enact and Observe

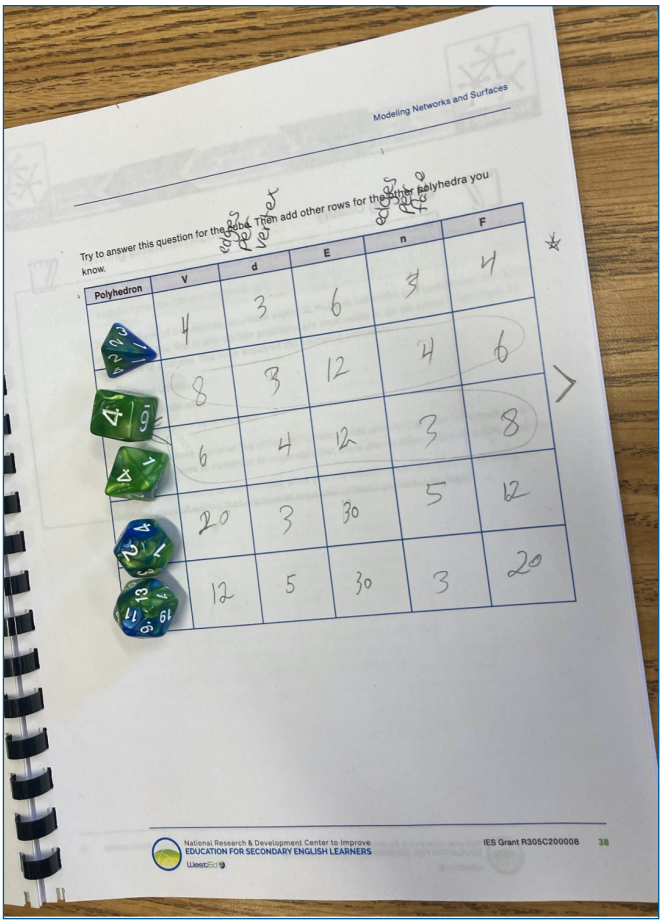
3. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered.
4. Monitor for students counting multiply, and offer them help to adjust for over-counting as related to degree or the number of faces to an edge.
5. Look for whether students are recognizing any of the dual relationships between Platonic solids (i.e., cube and octahedron, icosahedron and dodecahedron, and tetrahedron as self-dual).

Close and Connect

6. Ask students to share their work with a partner.
7. Have students complete the *Exploring Duality* handout:
   - Which pairs of Platonic solids seem most related to one another? Why?
8. Then, close with a whole class discussion using the following question:
   - What connections do they see between the original tessellation problem in the plane and the existence of Platonic solids?

Student Work Samples

Students noticed patterns across platonic polyhedra.



Writing Extension Activity

Students will construct arguments for why certain problems are impossible or why they have found all possible cases of something.

Purpose and Rationale

The purpose of this task is for students to construct arguments of proof by exhaustion or by impossibility.

Prepare

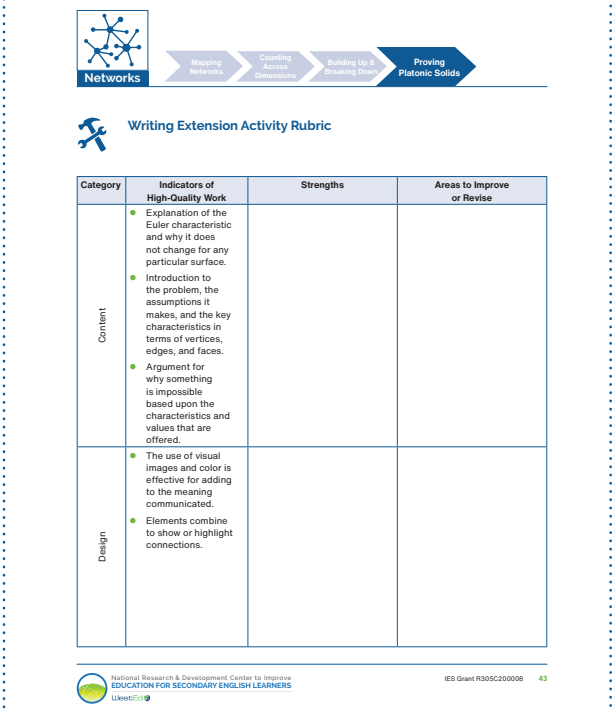
1. Have students turn to the *Writing Extension Activity Rubric* in their binders.
2. Go over the sections covering the *Content and Design* expectations.

Enact and Observe

3. Have students develop their writing responses around the following:
   - Why certain problems are impossible to do on a flat plane or the surface of a sphere.
   - An argument that there are no other Platonic solids possible.

Close and Connect

4. Have students share ideas they are developing to help each other refine their draft.


Networks
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Writing Extension Activity Rubric

Category	Indicators of High-Quality Work	Strengths	Areas to Improve or Revise
Content	• Explanation of the Euler characteristic and why it does not change for any particular surface. • Introduction to the problem, the assumptions it makes, and the key characteristics in terms of vertices, edges, and faces. • Argument for why something is impossible based upon the characteristics and values that are offered.		
Design	• The use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

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Writing Extension Activity Rubric

Category	Indicators of High-Quality Work	Strengths	Areas to Improve or Revise
Content	• Explanation of the Euler characteristic and why it does not change for any particular surface. • Introduction to the problem, the assumptions it makes, and the key characteristics in terms of vertices, edges, and faces. • Argument for why something is impossible based upon the characteristics and values that are offered.		
Design	• The use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

Reflect After Day 4

After the last activity of *Day 4: Proving Platonic Solids* has concluded, please reflect on student learning by responding to the following questions:

1. What connections were students making with the different polyhedra? To what extent were they able to generate arguments for why certain parameters are bounded or symmetric?

2. What additional models of language would be helpful for students as they complete their writing activities?

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