National Research & Development Center to Improve
EDUCATION FOR SECONDARY ENGLISH LEARNERS
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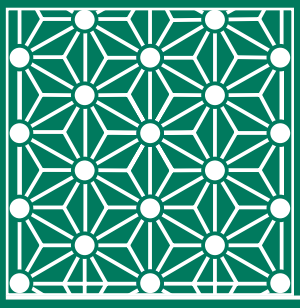


TEACHER MANUAL

Patterns of Growth and Change

Module Overview

This module challenges and supports students as they explore patterns occurring in a variety of settings and mathematical representations, including real-world situations, geometrical configurations, numerical sequences, and algebraic formulas.

Conceptual Understandings	Mathematical Practices	Language Practices
• Patterns can be described in terms of growth or parts. • Patterns can be represented concretely, numerically, or algebraically. • Descriptions that attend to components more easily correspond to closed forms.	• Make sense of problems and persevere in solving them. (MP1). • Look for and make use of structure (MP7). • Look for regularity in repeated reasoning (MP8).	• Describe sequential changes in numbers or objects. • Decompose arrangements into parts. • Coordinate and connect numbers, objects, and rules.

Overview of Content and Objectives

The arrow below displays how students will begin by exploring the multiple meanings of "pattern" and connecting patterns that focus on repetition with those that involve change. The activities in this module deepen students' sophistication in linking different parts of mathematical representations while also understanding how some approaches to looking for patterns may reveal underlying structure or regularity. The activities also facilitate student learning of key analytical practices, including the Standards for Mathematical Practice listed in the above table. Sequences of activities are offered to students as they extend their language use around description, a key practice in mathematics.

	Patterns of Repetition	Growing Sequences	Naming Numerical Sequences	From One Step to the Next	Describing Growth and Change
Content objectives: Students will	• Describe patterns in terms of symmetry or repetition. • Identify different kinds of patterns and articulate attributes of categories.	• Describe how terms in a sequence change. • Use changes to categorize and extend sequences.	• Connect different parts of numerical, geometric, and symbolic sequences. • Link sequences that are related geometrically, numerically, and symbolically.	• Solve story problems that have recursive patterns. • Decide whether patterns have linear or exponential type growth.	• Generate new sequences and problems • Solve problems involving sequences using a variety of techniques
Tasks at a Glance	• Math and Me (Self- Reflection) • Novel Ideas Only • Sort and Label • Classmate Bingo • Patterns in the Physical World • Describe and Order	• Introducing Sequences • Reading in Four Voices • Describe and Order: Expert Group • Individual Matrix • Expert Matrix	• Counting Handshakes Problem • Step it Up • Read and Connect: Formulas	• Patterns of Growth • Problem Solving with a Partner	• Flipping Triangles • Counting Paths • Writing Extension

Key Considerations

How to Group Students

In many of these activities, students will work with a partner or in a small group. There are many ways to group students into teams. Students can be grouped by interests, mathematics ability, compatibility, or convenience. There are potential advantages and drawbacks with different ways to group students. For example, convenience grouping, when a teacher asks students who sit near each other already to work together, is easy to organize, but student groups may not have students with different interests or abilities. Grouping students with a variety of interests or abilities can bring together student strengths, but these groups may be harder to manage. Consider different ways to group students before each activity, and intentionally choose a grouping strategy.

Some extended tasks, such as the Jigsaw Project, will offer multiple groupings and regroupings that will enable students to develop expertise and present to their peers.

Attention to Language Development

The following list of interactive classroom tasks details how students are supported as they develop language practice around description. As students develop greater sophistication in describing for mathematical purposes, there are various approaches that they may take.

They may begin with a "laundry list" approach in which they list details serially without a greater organizational scheme. As they gain more experience, they make more sophisticated descriptions, such as: 1) those that are organized more sequentially with a "step-by-step" structure; or 2) those that attend to part-whole and whole-part relationships. In mathematics, the descriptions that are the most productive are those that highlight or bring to the foreground numerical, geometrical, or other relationships, often across representational families (such as equations and objects).

- ***Sort and Label:*** In small groups, students describe images, group them, and create labels for those groups.
- ***Describe and Order:*** In small groups of four, students describe their individual images without showing them to others. Together, the group decides on an order for the images. Once they have reached agreement, they reveal their images and check to see if they agree.
- ***Describe with Guidance Card:*** With a partner, students choose from selected actions and formulaic expressions in order to describe a card. They prepare to share their description with another pair. They write a set of bullet points to guide their presentation.

- ***Reading in Four Voices:*** In a small group of four, students read written descriptions of a growing pattern of square frames. They decide upon quality features of a description to incorporate into their own work with different patterns.
- ***Revise a Description:*** With a partner, students revise one of the descriptions from the previous task. As they revise, they draw upon a set of "moves", such as adding detail, adding framing or signposts, deleting unnecessary information, connecting existing parts.
- ***Individual and Expert Matrix:*** In small groups of four, students work on understanding one sequence, specifically how it grows and its key characteristics. Students work individually to generate a description of the pattern that will enable others to reproduce it. They confer with others in their expert groups to reach a consensus about what is a clear description of their pattern and answers to focus questions that generally apply to patterns of growth.
- ***Read and Connect – Formulas:*** In a small group, students connect the different descriptions that they have worked with before with algebraic formulas. They attend to how elements and parts of the description fit with different parts of different formulas and weigh the advantages of different approaches.
- ***Writing Extension Activity:*** Students independently create a written description of a pattern of change. Peers critique and offer suggestions to each other's descriptions before students individually revise.

As students engage in these activities, they are challenged and supported in their language development along three dimensions:

- They grow from more interactive, dialogic interactions to negotiated descriptions that develop into more monologic, edited, and rehearsed descriptions.
- Students move from working together with peers to figure something out toward becoming more expert and authoritative in their uses of language.
- The language that students use to refer to the objects that they are discussing moves from more everyday uses toward more technical uses, appropriate for developing deeper mathematical ideas and practices.

How to use the Teacher Materials

The following manual presents the purpose and rationale of each student activity as well as how to prepare for using the activity with students, how to enact it, and how to close and connect what students learn in our activity to their developing understanding. Please review these materials and complete each student activity before you begin.

The materials also present three examples of student talk and work. The examples are included as sample student work, student dialogue, and vignettes that demonstrate student participation in the activities. Review the sample student work for students' unique solution strategies. Consider students' mathematical understandings that are evident in these activities. These examples are not designed to be shared with students as "right answers." Give hints strategically so that students do the thinking and problem solving.

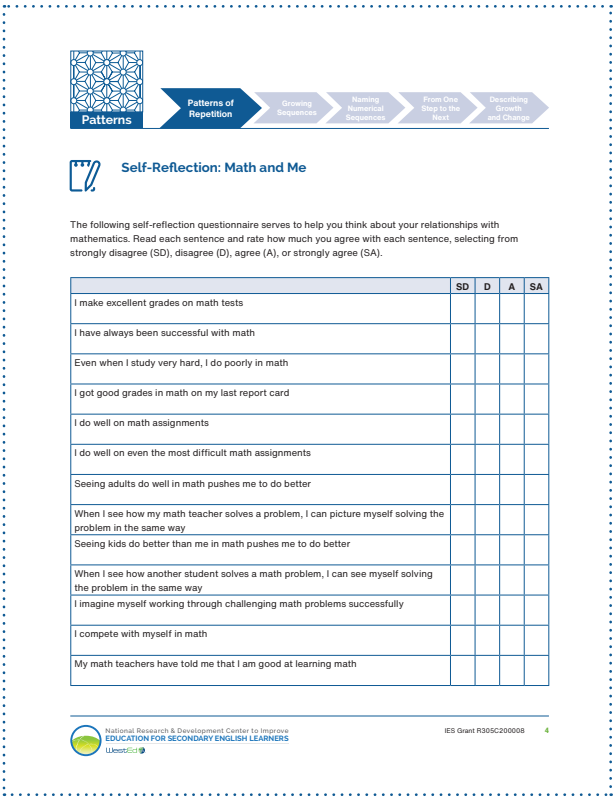
TEACHER NOTES

The main purpose of Day 1 is to draw upon students' prior experiences with different kinds of patterns. The main theme of the first day is to focus on patterns where there are some elements that are repeated over and over again. This focus will shift in subsequent days as students consider patterns of growth or change.

When students share their ideas about a topic before formal instruction, they can begin to make connections from their prior knowledge to the new skills and ideas, and teachers can use the students' ideas and the language they used to share them to inform their instruction.

Self-Reflection: Math and Me

This self-reflection activity provides students with an opportunity to reflect on themselves as math learners and provides teachers with some insight into students' self-perceptions.



Purpose and Rationale

The purpose of this activity is to capture students' self-reflections on and relationships to mathematics.

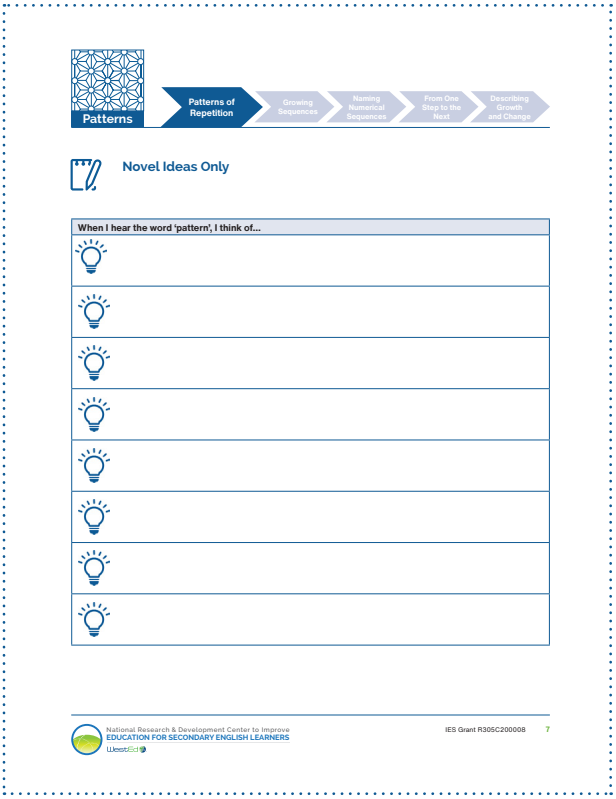
How students see themselves as learners can inform their actions and practices during mathematics class. As we understand our students, their strengths and weaknesses, as well as their perceptions of themselves, we can better facilitate student learning and respond in ways that support student growth.

Enact (what to do)

1. Tell students the importance of their own perceptions and attitudes and how you are interested in learning more about them as mathematics learners.
2. Distribute the *Self-Reflection: Math and Me* handout to each student.
3. Read the directions aloud. Ask students to work independently and be honest as they complete the self-reflection.
4. Collect all handouts and tell students that they will do a similar activity at the end of the program.

Novel Ideas Only

The *Novel Ideas Only* task elicits what students already know about patterns and emphasizes the valuable prior knowledge that they bring to this unit..



Purpose and Rationale

The purpose of this activity is to surface students' prior knowledge and experiences regarding patterns.

This activity provides an opportunity for students and the teacher to hear students' current ideas about patterns and to connect to students' ideas of patterns before inviting them to expand their skills and knowledge in subsequent activities. The small group structure provides an opportunity for students to share their ideas with a small group of peers; then, in the full group, students hone their listening skills as other groups present. As you enact this activity, be ready to listen carefully for student ideas that you may want to highlight and encourage during later activities. Connecting later learning to students' current ideas can help students make connections and build a robust understanding of patterns.

Prepare

1. Before you begin, note at least three ideas that you anticipate your students will use to complete the prompt: "When I hear the word "pattern," I think of ...". Note your ideas here:
   -
   -
   -
   -

2. Review both parts of this activity and determine how you will group students. Students will be working together for about 10 minutes.

Enact (what to do) and Observe (what to look/listen for)

3. Distribute the *Novel Ideas Only* handout to each student. Read the directions for small group work aloud.
4. Give students 3 minutes for small group work. In each group, one student will share an idea in response to the prompt, another student will repeat it, and all students will write it down. Students continue sharing, repeating, and writing ideas for 3 minutes.
5. Note that everyone in the small group should have the exact same information on their papers.
6. After students work in groups for 3 minutes, lead a full-class sharing where students report out their "novel ideas." Ask one student from one group to stand and read all the ideas that their group collected. Other students should listen carefully and fill in the lightbulb if one of their ideas is read aloud. Then, the next group should read aloud only the ideas from their group that were not yet shared. Continue this process until all groups have shared. To assist in keeping track of what groups have shared, ask groups to sit down after they have shared their novel ideas.
7. After students have completed reporting, encourage them to keep listening and to add to their list if they hear a new idea that they would like to include.
8. Listen for one or two repeated ideas shared by students or a theme across student reporting that you can use during the lesson closing.

During small group work:

- Encourage students to listen carefully to their classmates' ideas.
- Prompt students to take turns sharing and repeating ideas.
- Pause the activity after 3 minutes, as this is enough time for students to write their initial ideas.

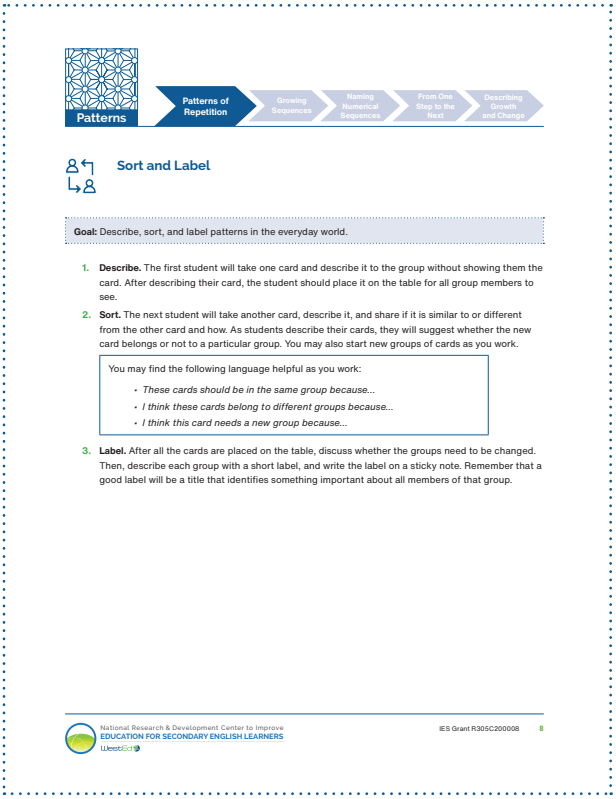
Close and Connect

9. Close the activity by thanking students for reporting out their novel ideas and then sharing a theme that you heard across student responses; for example, how patterns have sequences that repeat, are visually appealing, or can be in nature. Highlight language that you have heard students use that will be useful in later activities.
10. Then, tell students that in the next activity they will look at and describe different visual patterns and that they should think about how those new patterns connect with the ideas they have shared so far.

Remember that the goal of the activity is to collect students' ideas and encourage their thinking; the goal is not to write out an exhaustive list of all ideas about patterns. Refrain from asking students to write down all the ideas they hear.

Sort and Label

There are different types of repeating structures and a variety of mathematical relationships present in patterns in everyday contexts. In this activity, students describe patterns that they may have encountered in real-world contexts and then work together to sort different types of patterns, identifying and naming the attributes that define them.



Purpose and Rationale

The purpose of this task is for students to identify patterns by describing common features and identifying key attributes.

The *Sort and Label* activity is an opportunity for students to share what they notice in the different patterns in everyday objects by describing attributes and characteristics of different visual patterns. As they describe patterns to other students in their small group, students will practice identifying key attributes of patterns and justifying how to sort the patterns based on a certain attribute. They will also practice making more sophisticated descriptions as they categorize and group patterns. The pattern cards emphasize different ideas of symmetry and repetition, some that may be new to students and others that they may have seen before. While students are working, listen for the language that students use to describe and categorize the patterns. In discussion, highlight ideas of symmetry and repetition as well as other attributes that students emphasize.

Prepare

1. Before you begin, review the cards. Identify different categories that students may sort the cards into and note cards that would be in that category. List three or more possible categories below:
   -
   -
   -
2. Students will work in a group of four.

Enact and Observe

3. Organize students into groups of four. Distribute **one set of cards** to each group and the Sort and Label handout.
4. Read the directions for the task aloud. Tell students that they will sort the cards one at a time, discussing their categories as they work. On each turn, a student will take a card and describe it to the group without showing them the card. After describing their card, the student should place it on the table for all group members to see. In subsequent turns, students will describe their cards and suggest whether the new card belongs or not to a particular group. Students may also start new groups of cards as they work.
5. While students are working, make note of how they are sorting the cards, and prompt student conversations using the sample formulaic expressions.
6. If students ask you for help when sorting cards or labeling their groups, encourage them to note the similarities and differences across cards in the current groups and to add additional descriptions to sorted cards.
7. After all the cards are placed on the table, students can discuss whether the groups need to be changed. Then, when the students have sorted all the cards, ask them to describe each group with a short label. Write the label on a Post-It note.

Share these formulaic expressions for students to use during small group work:

- *These cards should be in the same group because ...*
- *I think these cards belong to different groups because ...*
- *I think this card needs a new group because ...*

Close and Connect

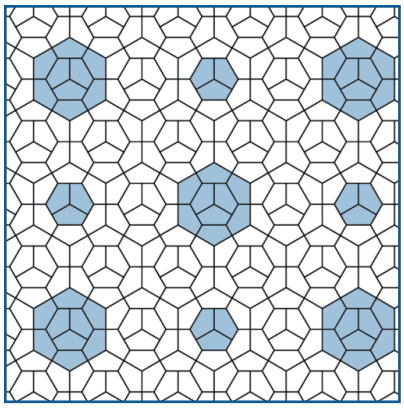
8. Close the activity by sharing a sample of categories that were present across groups. Highlight ideas of symmetry and repetition.
9. Then, tell students that in the next activity they will learn more about why there are so many patterns in everyday life.

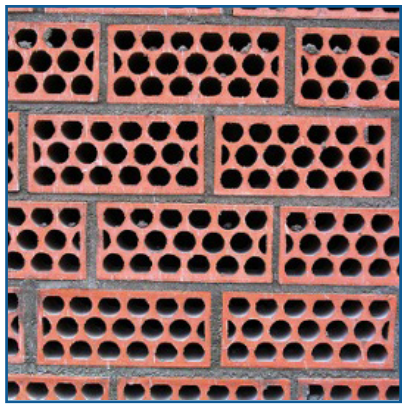
Sample Student Work and Sample Labels

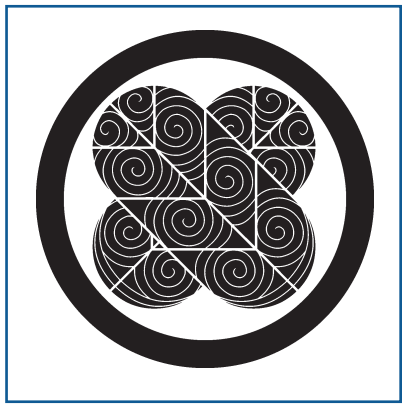
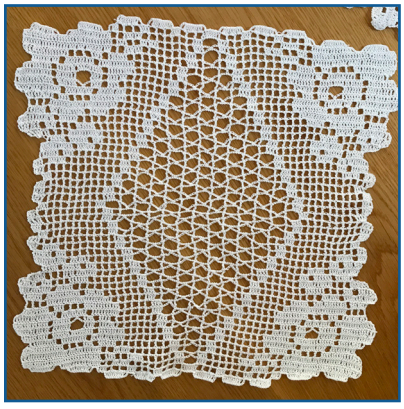
Group	Labels				
1	Shapes	Tiles	Flowers		
2	Handmade	Architectural	Computer generated		
3	Carpet	Flowers	Covered patterns	Holes	Tiles
4	Honey	Forest	Reverse	Hexagon	Square
5	Shapes	3D shapes	Fabric		
6	Circular	Rectangular	Star (polygon)	Lines	
7	Nature	Flat Designs	Floors		

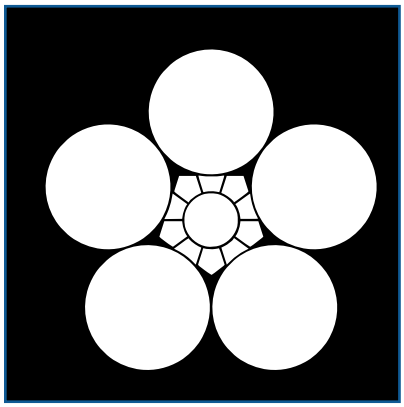
Table 1. Student-Generated Categories for Patterns

Sort and Label: Pattern Cards

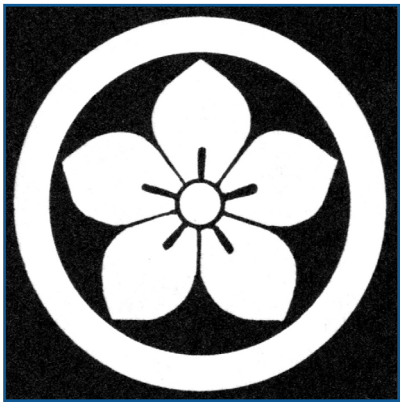




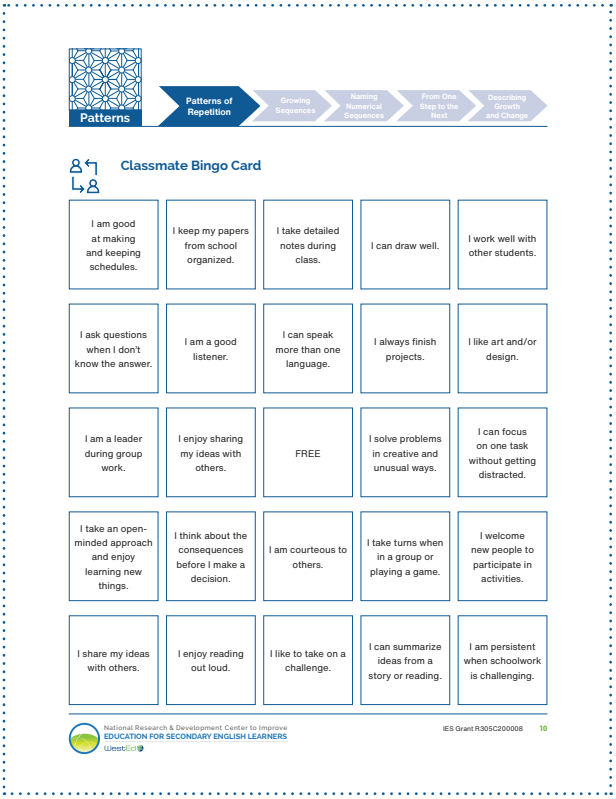





Classmate Bingo Card

How students see themselves as mathematics learners and as students in general affects their readiness to learn and their overall success. Also, how students see others can influence how they react to them, how they work together, or how they take up others' ideas. The *Classmate Bingo Card* activity can be used to recognize and celebrate the diverse sets of skills and competencies that students bring to the mathematics classroom.



Purpose and Rationale

The purpose of this task is to create an opportunity for students to learn about their classmates and the skills and strengths that they bring to the mathematics classroom.

All students have strengths that they may not recognize as relevant to the mathematics classroom. Highlighting the strengths that students bring to the math classroom will prepare them to engage in new activities as well as to see themselves as mathematics learners.

Prepare

1. Before you begin, read the skills listed and consider how these skills may be relevant to math class. Make notes on your copy of the *Classmate Bingo Card*.

Enact and Observe

2. Distribute the *Classmate Bingo Card* to each student. Read the directions aloud.
3. Give students **1–2 minutes** to read the skills listed independently. Then, give students **5 to 8 minutes** to complete the activity.

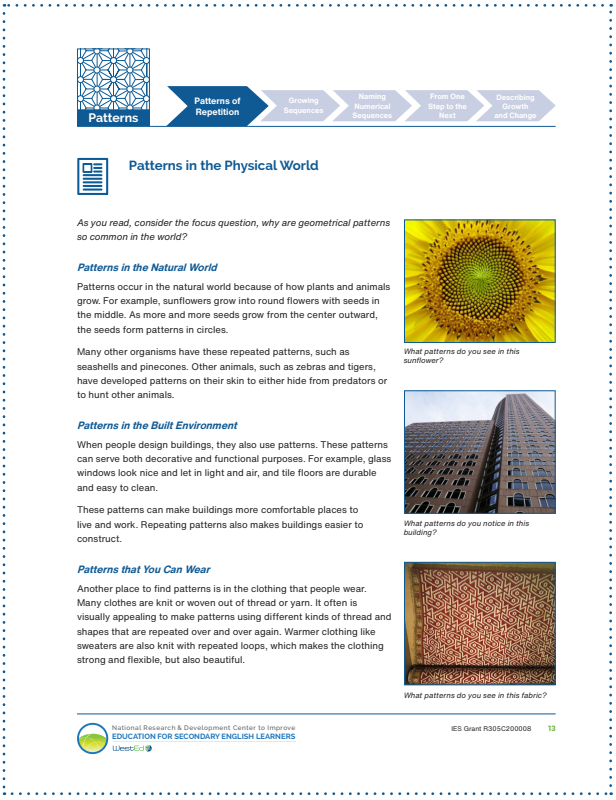
4. Prompt student talk. Encourage students to think of what they know how to do well both in and outside of school. Prompt students to ask each other questions, encouraging their classmates to identify their skills.

Close and Connect

5. After students have completed the bingo card, lead a classroom discussion for **5–8 minutes**. Ask students: *What did you learn about the skills that your classmates have?*
6. When students share their classmates' skills, make a connection to how that skill may be relevant to this class. For example:
    - A student who can draw well may be able to show their mathematical thinking in a diagram.
    - A student who always finishes projects may persevere when the task is challenging.
    - A student who speaks another language may have multiple ways of considering similar tasks.
7. Share with students that later they will explore a problem that is about handshakes—each exchange they had with another partner on the bingo card could be considered one handshake. Today, the next activity is a reading about patterns in the physical world.

Patterns in the Physical World: Why Are Geometrical Patterns so Common in the World?

Geometrical patterns occur both naturally and in the human-built world. As students identify where they see patterns and learn how patterns occur commonly in the physical world, they will make connections from patterns they have seen to their purposes as well as learn more about why patterns are common.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Patterns in the Physical World

As you read, consider the focus question, why are geometrical patterns so common in the world?

Patterns in the Natural World

Patterns occur in the natural world because of how plants and animals grow. For example, sunflowers grow into round flowers with seeds in the middle. As more and more seeds grow from the center outward, the seeds form patterns in circles.

Many other organisms have these repeated patterns, such as seashells and pinecones. Other animals, such as zebras and tigers, have developed patterns on their skin to either hide from predators or to hunt other animals.

What patterns do you see in this sunflower?

Patterns in the Built Environment

When people design buildings, they also use patterns. These patterns can serve both decorative and functional purposes. For example, glass windows look nice and let in light and air, and tile floors are durable and easy to clean.

These patterns can make buildings more comfortable places to live and work. Repeating patterns also makes buildings easier to construct.

What patterns do you notice in this building?

Patterns that You Can Wear

Another place to find patterns is in the clothing that people wear. Many clothes are knit or woven out of thread or yarn. It often is visually appealing to make patterns using different kinds of thread and shapes that are repeated over and over again. Warmer clothing like sweaters are also knit with repeated loops, which makes the clothing strong and flexible, but also beautiful.

What patterns do you see in this fabric?

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Purpose and Rationale

The purpose of this task is for students to deepen their background knowledge of patterns that arise in the physical world in contexts such as nature, construction, and textiles.

This activity provides students with information on why geometrical patterns occur in the natural and built worlds. Additionally, through taking turns and working with a partner, students practice discussing mathematical text using the *Clarifying Bookmark I*. The *Clarifying Bookmark I* provides ideas on what to say, ideas that students can use in this and future discussions.

Prepare

1. Make sure to decide how you will group students for this activity.
2. Read the text about geometrical patterns. As you read the text, jot down some ideas about where you think students will be clarifying for each other through discussion.

Enact and Observe

3. Organize students into groups of two and distribute a copy of *Patterns in the Physical World* and the *Clarifying Bookmark I* to each student. Review the directions. Tell students that they will each take a turn reading a section of the text.

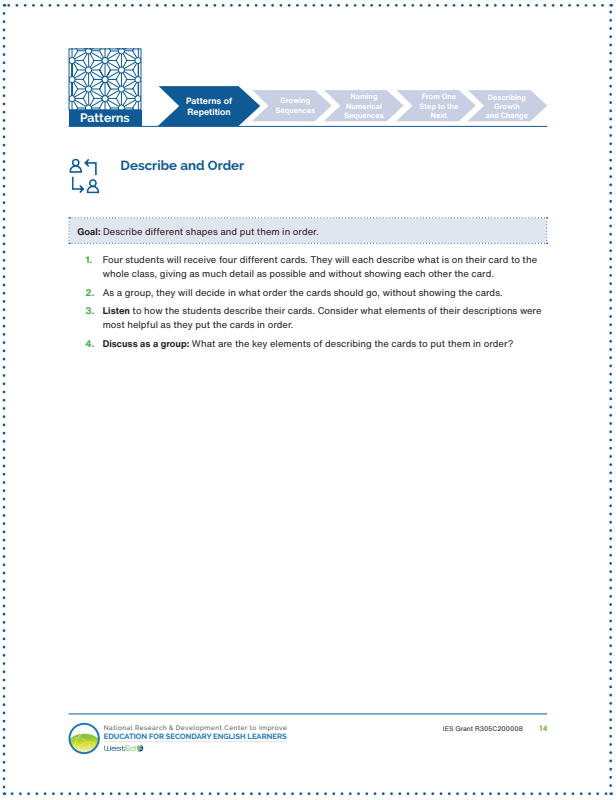
4. After the first student reads, the student will choose a strategy from the *Mathematics Clarifying Bookmark* I and use one of the formulaic expressions to share ideas; then, the second student will also choose a formulaic expression from the Mathematics Clarifying Bookmark to share ideas. Students will alternate reading and continue taking turns until they have finished reading.

Close and Connect

5. Close this activity by asking some students to share with their partner one new idea that they learned. Encourage students to share with the whole class what their partner said.

Describe and Order

The *Describe and Order* activity provides an opportunity for students to practice describing a term with the goal of ordering the terms without viewing them first. Four students will order a set of cards as a means to model the activity and set up opportunities to draft a description of how the shapes change.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Describe and Order

Goal: Describe different shapes and put them in order.

1. Four students will receive four different cards. They will each describe what is on their card to the whole class, giving as much detail as possible and without showing each other the card.
2. As a group, they will decide in what order the cards should go, without showing the cards.
3. **Listen** to how the students describe their cards. Consider what elements of their descriptions were most helpful as they put the cards in order.
4. **Discuss as a group:** What are the key elements of describing the cards to put them in order?

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Purpose and Rationale

The purpose of the *Describe and Order* activity is for students to learn how to order terms in a sequence that is growing. Today, four students will model the activity for other students as this will prepare all students for doing the activity later as well as provide a common set of shapes for all students to describe.

Through describing their cards to others, students will see and say key elements and generate and exchange new ideas about what to look for when trying to order terms in a sequence. They will also engage in detailing more sophisticated descriptions, seeking to make sure that the students in their group understand their term and place it in the sequence correctly. Students will also learn more about describing sequences as they listen to how others describe the other terms in the sequence.

Prepare

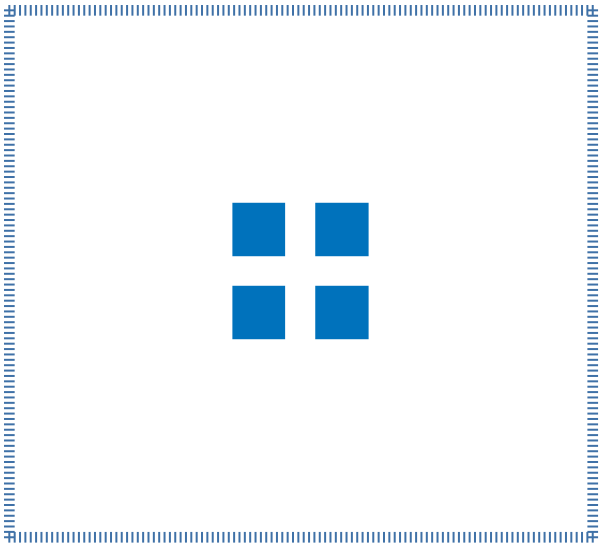
1. Identify four students to model this activity.
2. Cut out the five cards for this sequence.
3. Review the set of cards and describe the terms out loud to prepare for facilitating students as they describe and sequence the terms.

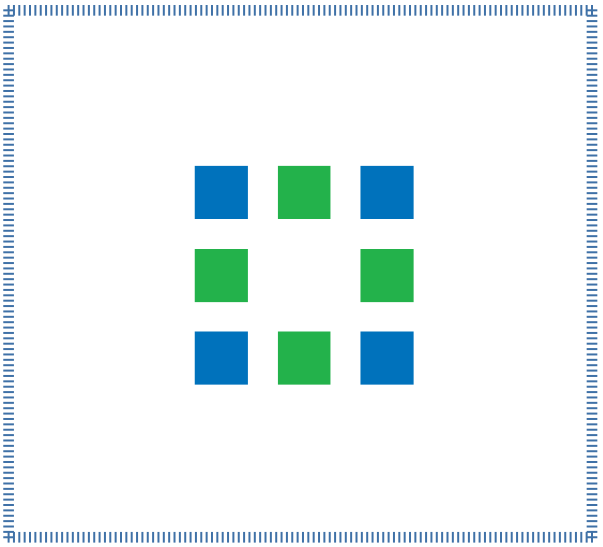
Enact and Observe

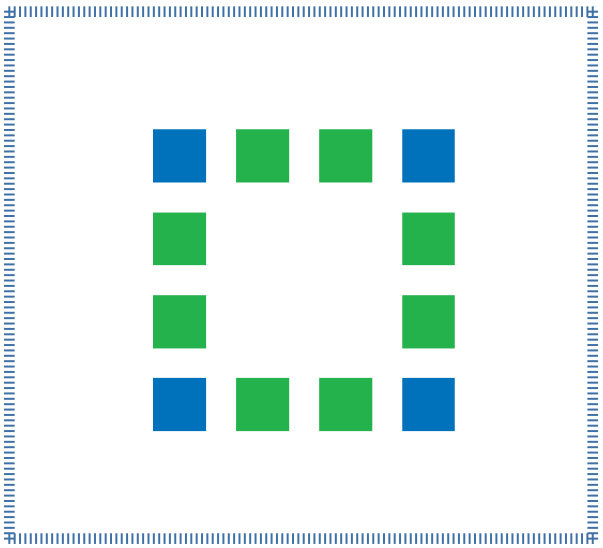
4. Tell students that four of their classmates will be modeling an activity that they will do later in the week.
5. Instruct the four students who will model the activity to sit together in the middle of the room while other students watch. Each student who is modeling the activity will take out one card that shows a term in a sequence.
6. Instruct students not to show their cards to others. Explain that they will describe their card to their group members, giving as much detail as possible and without showing the card. Students will then discuss how to order the cards without revealing them yet. As students agree on a sequence of the cards, they will place the cards face-down on the table. Once everyone has agreed on the order, students will flip the cards over to see if the order makes sense.
7. As the four students are working, prompt them to provide enough detail and to listen carefully to their peers. Encourage students to ask each other questions as they are describing and making connections from their cards to other cards.
8. Have the four students reveal the order of the cards, and make sure that all students see the ordered cards.

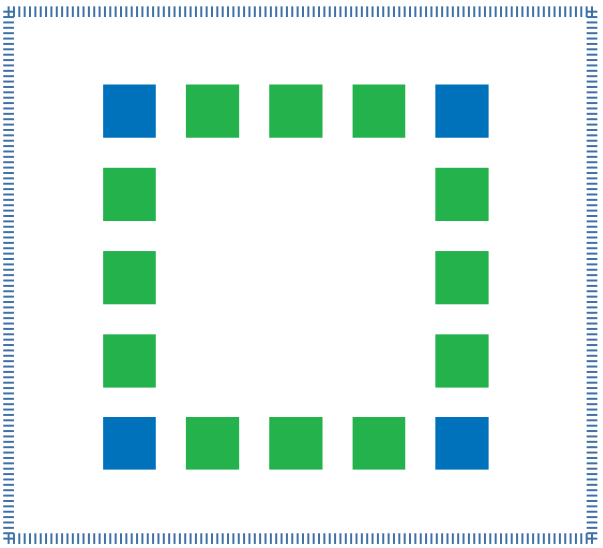
Close and Connect

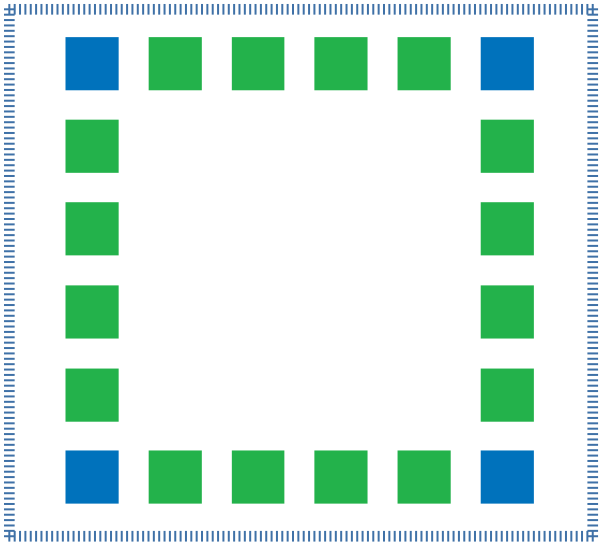
9. Ask the full group to reflect on what kinds of information helped the most as they were trying to describe and sequence. Ask students to consider: What are key elements of describing the cards to put them in order?
10. As this is the last activity on Day 1, thank students for sharing their *Novel Ideas Only*, sorting and labeling the pattern cards, participating in *Classmate Bingo*, discussing geometrical patterns with their partners, and describing and ordering the different shapes. These activities all sought to activate our current ideas about patterns—on Day 2, the focus will turn from patterns of repetition to patterns of change.
11. Prompt students to complete the *Daily Writing Prompt*. The prompt asks students to draft their own description about how the shapes from the Describe and Order activity are changing. Students should also identify the strengths of their own description.

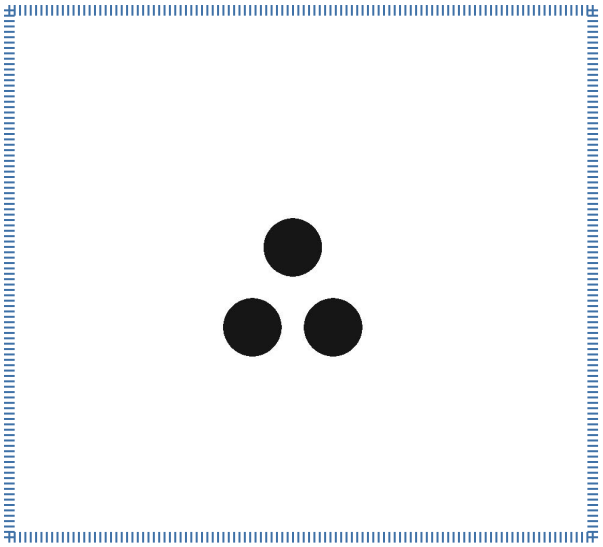


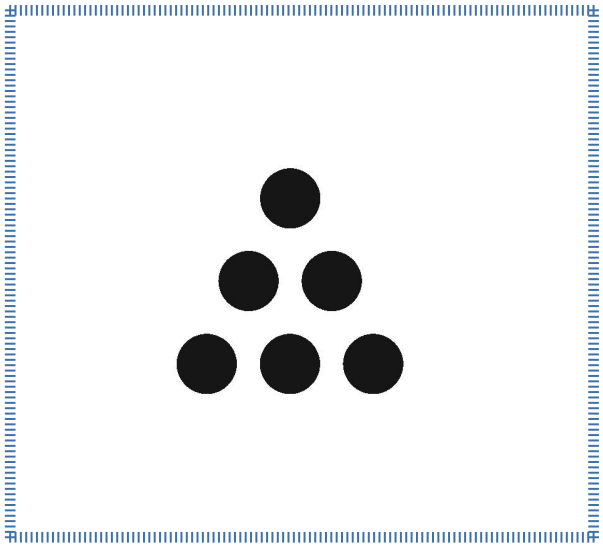


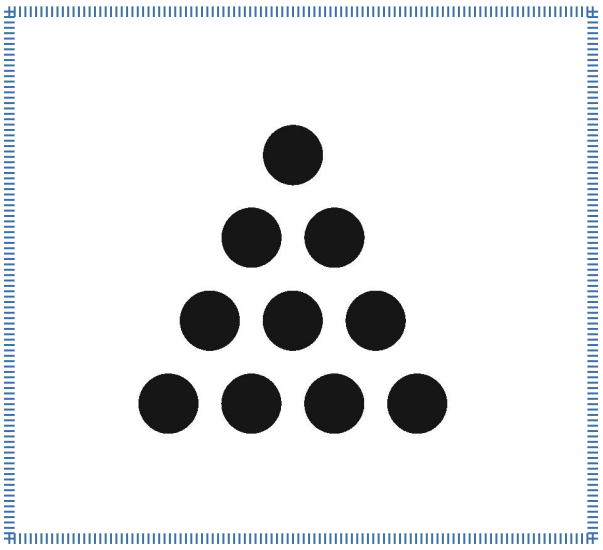


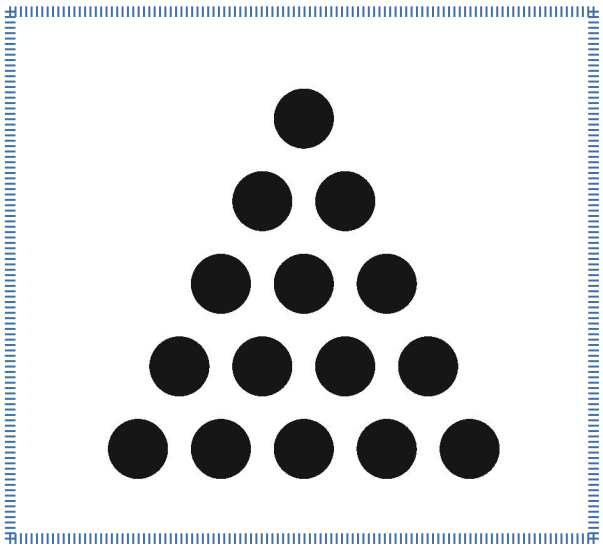


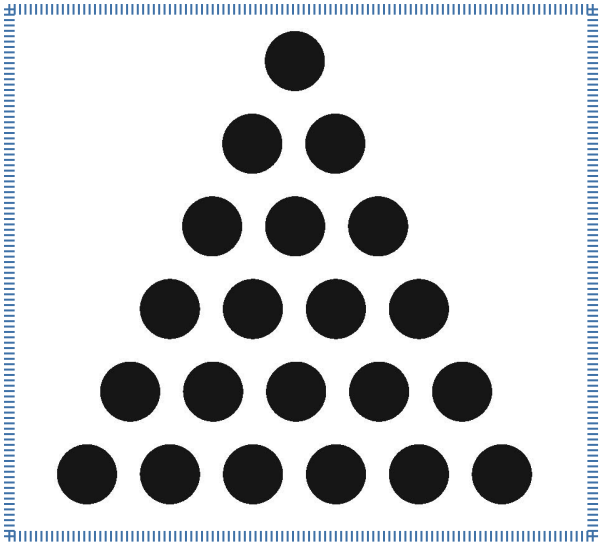


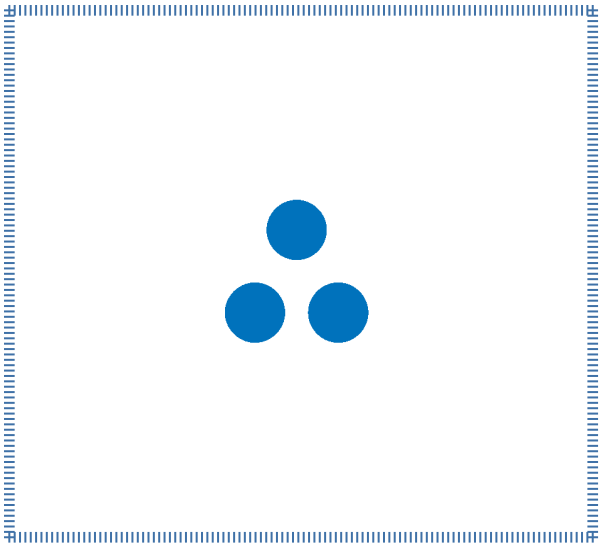


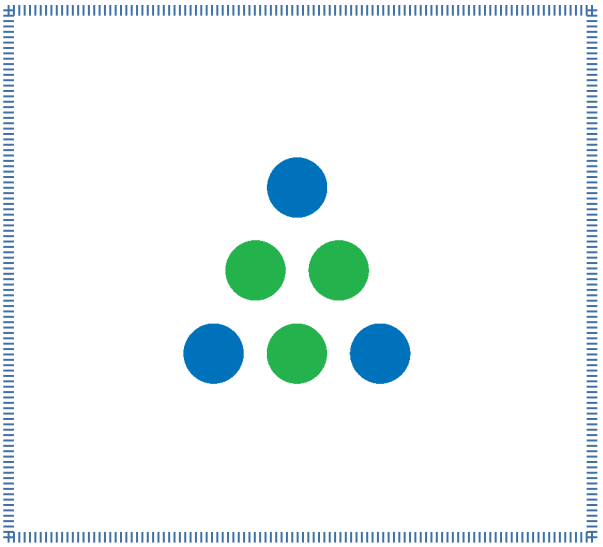


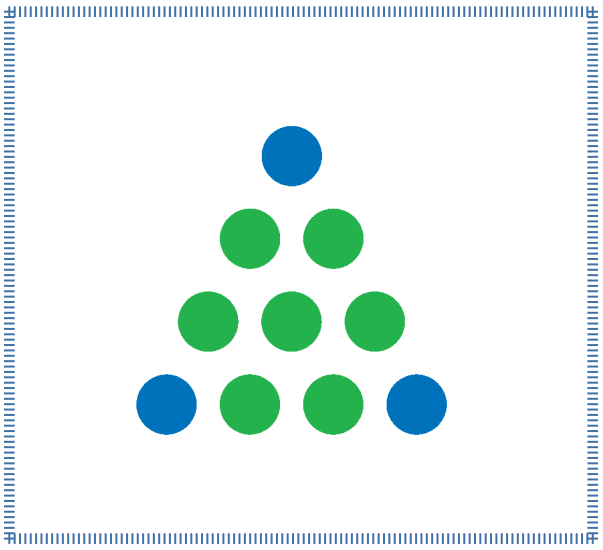


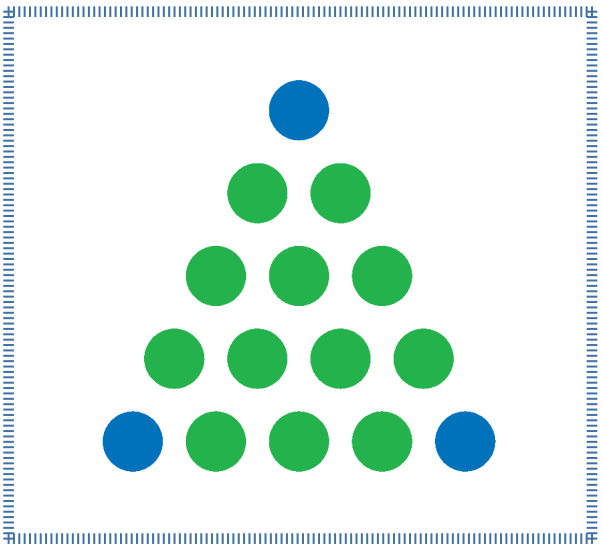


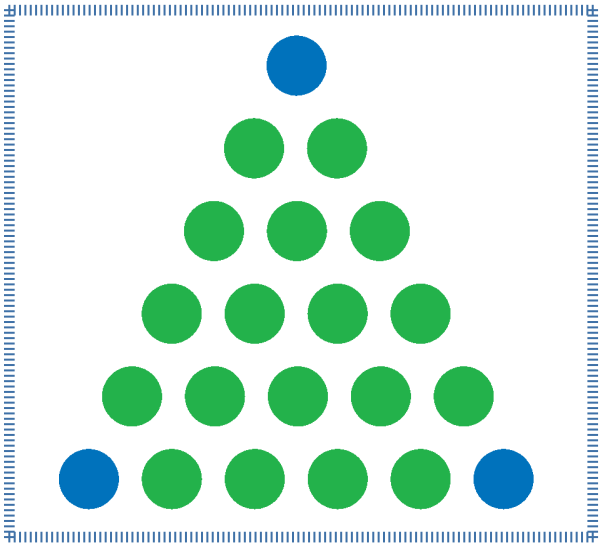


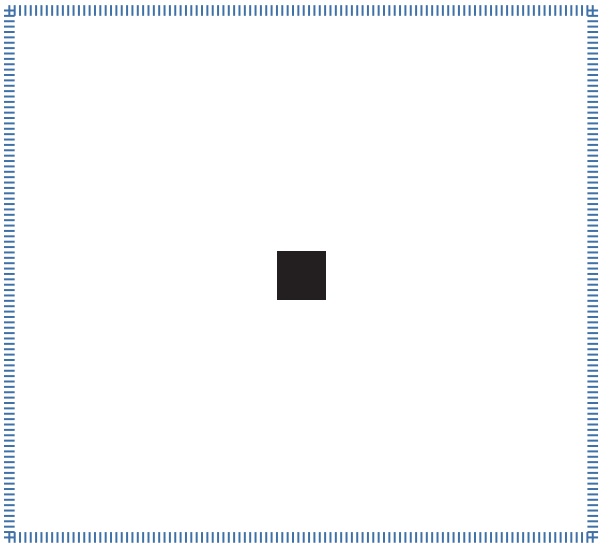


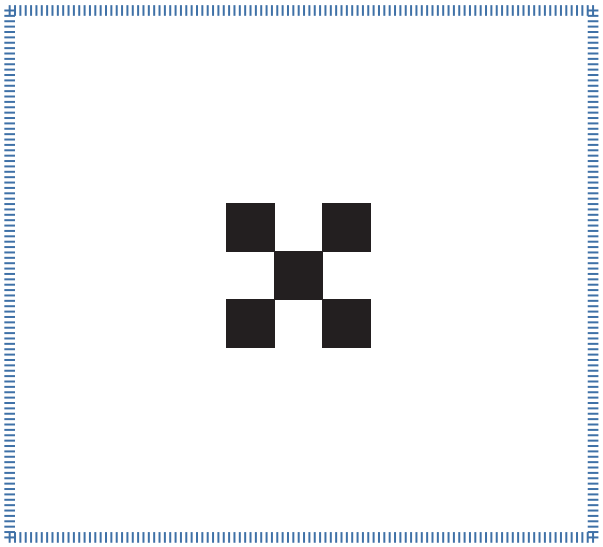


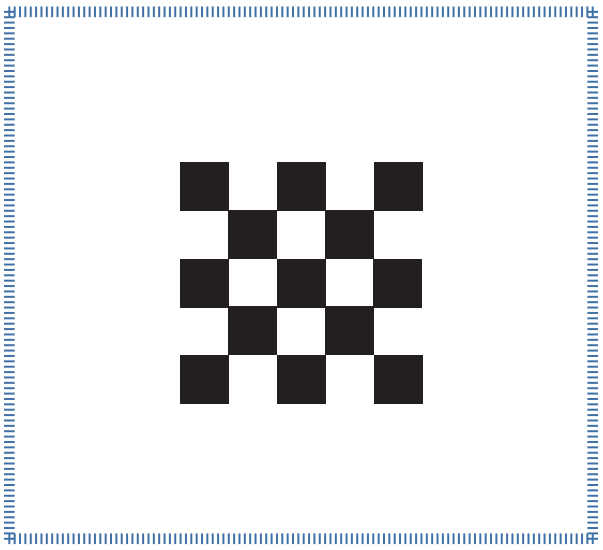


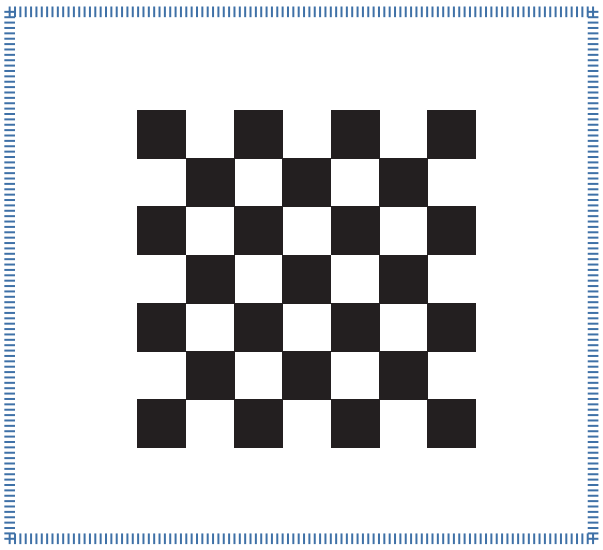


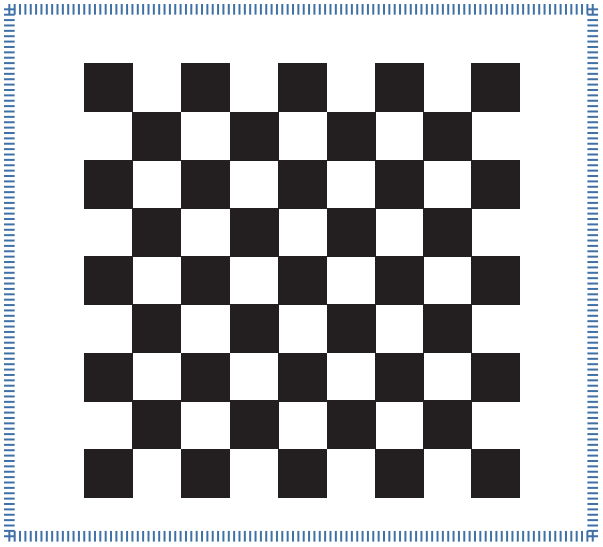


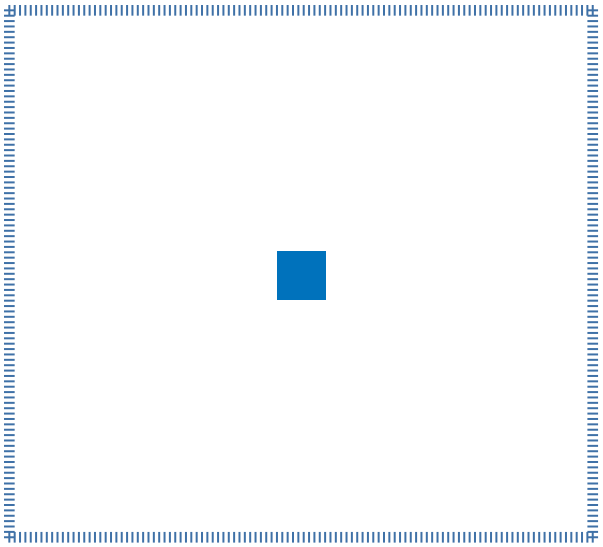


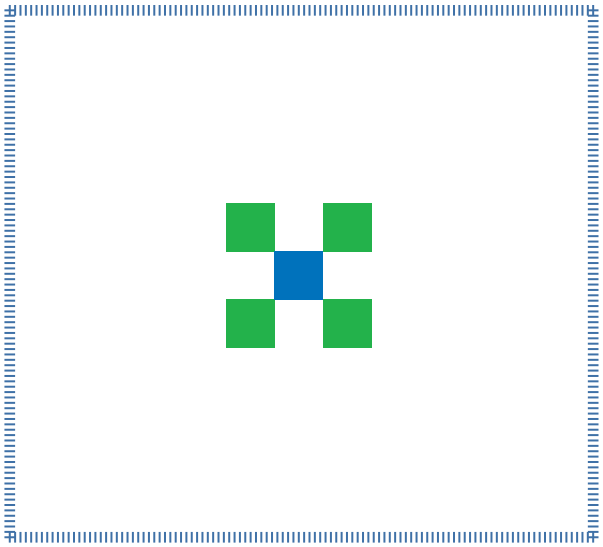


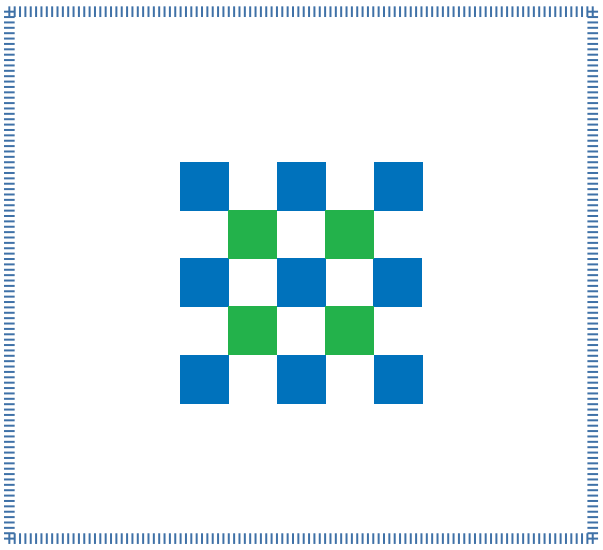


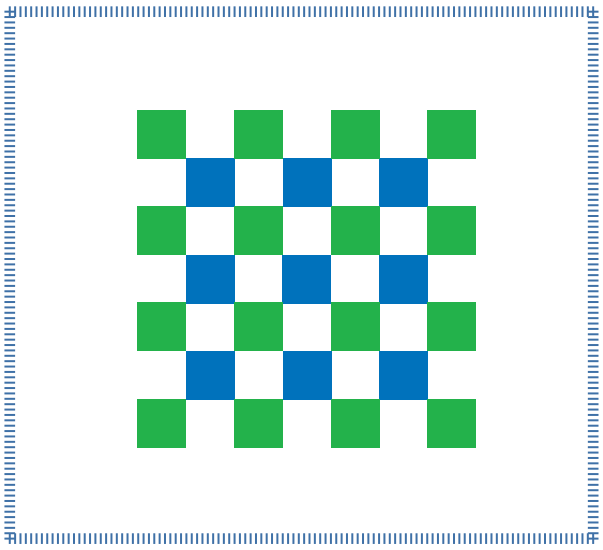


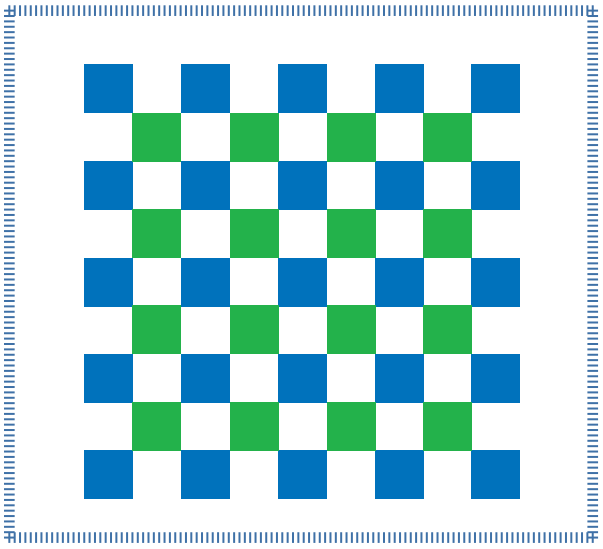


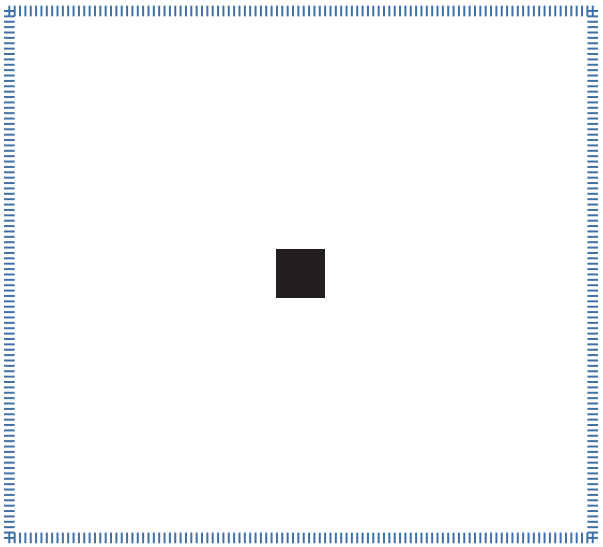


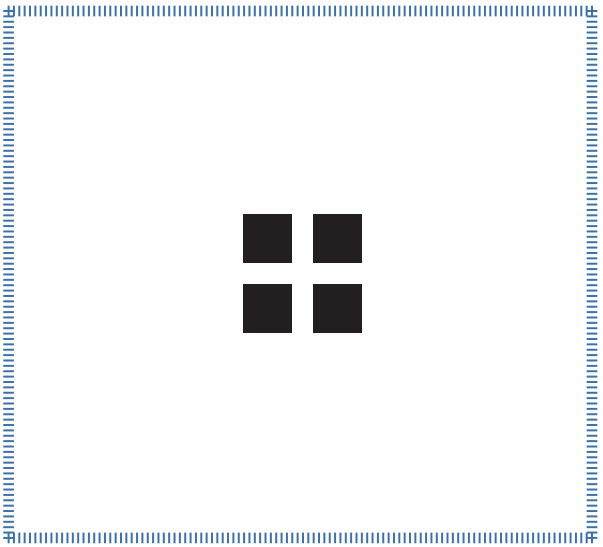


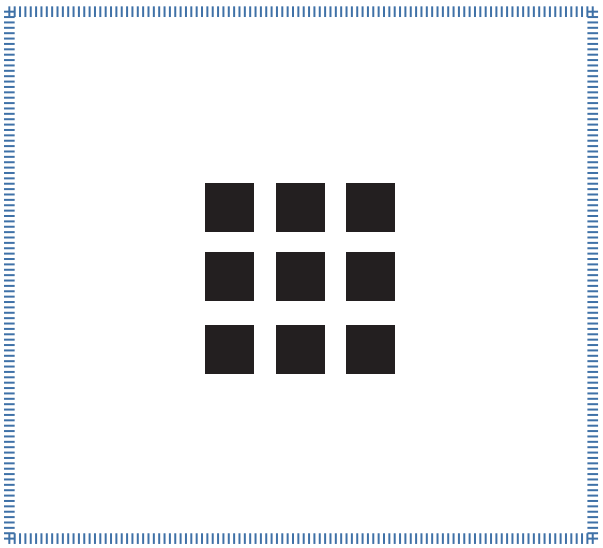


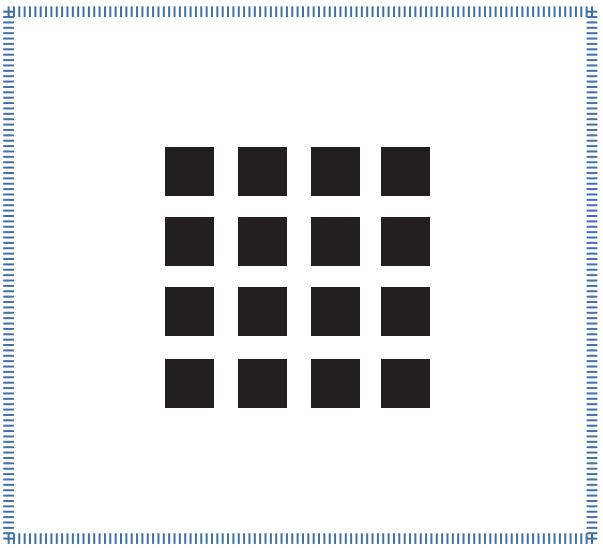


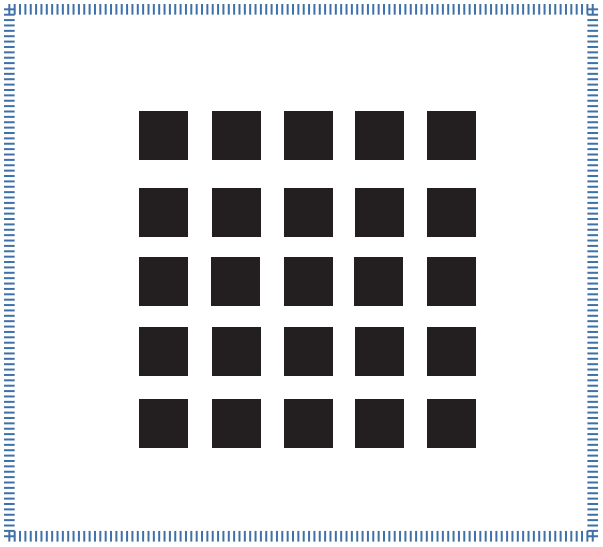


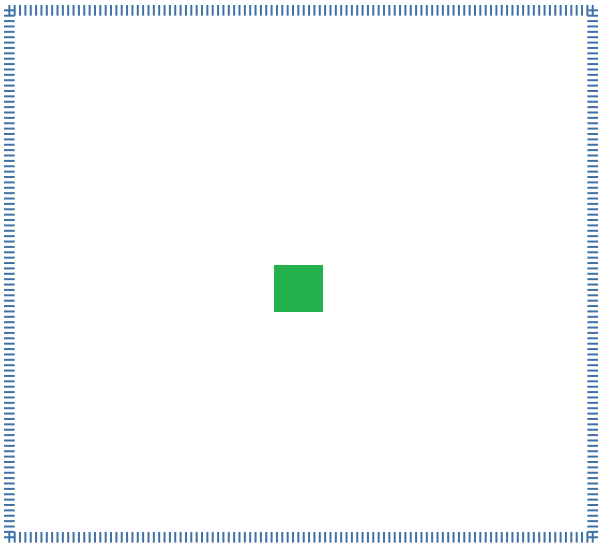


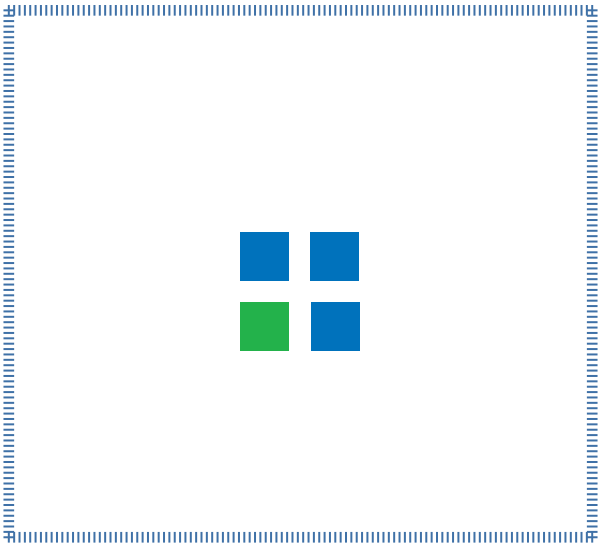


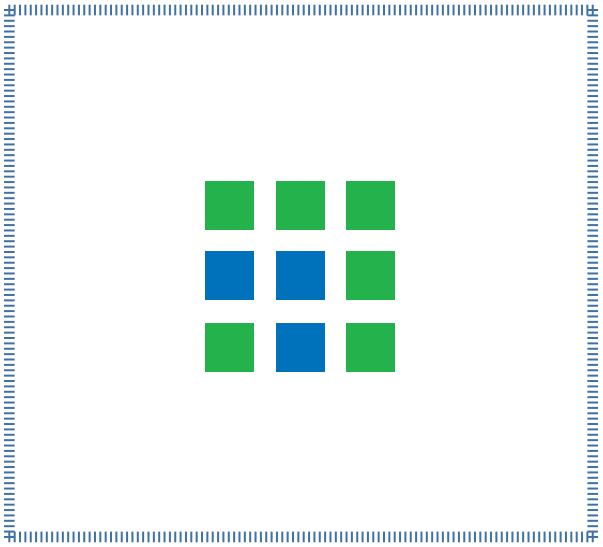


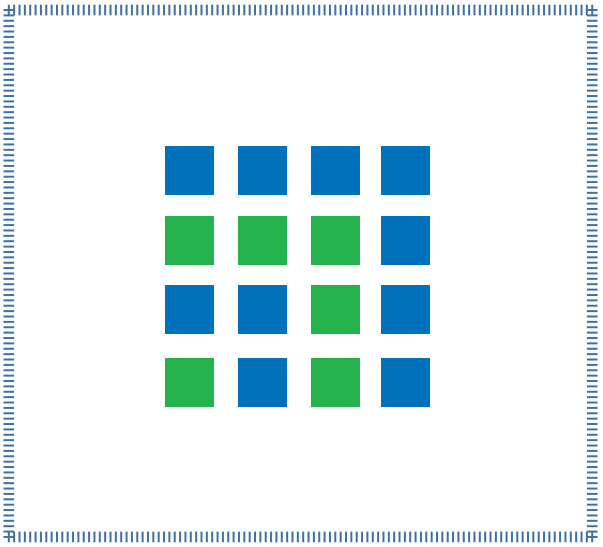


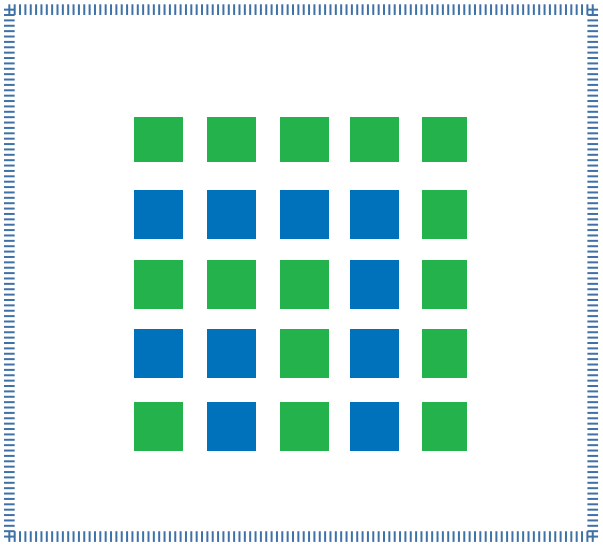


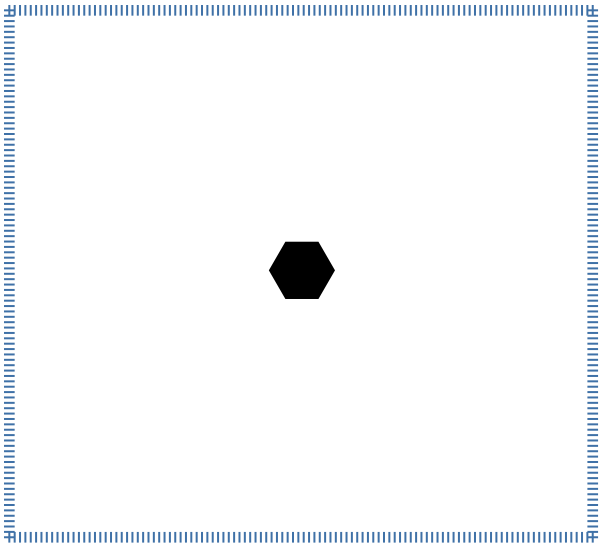


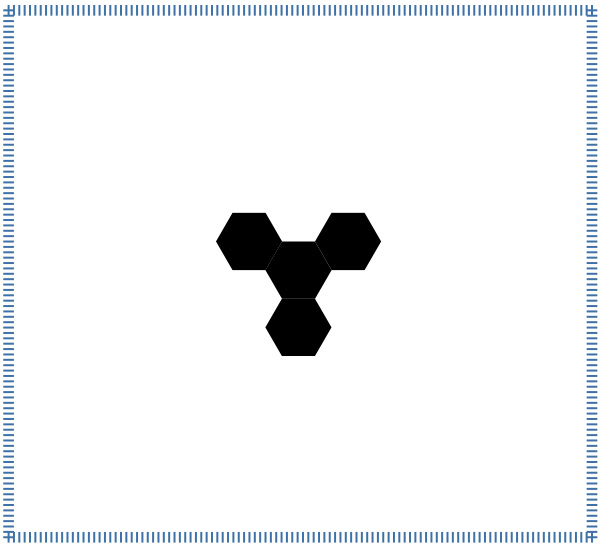


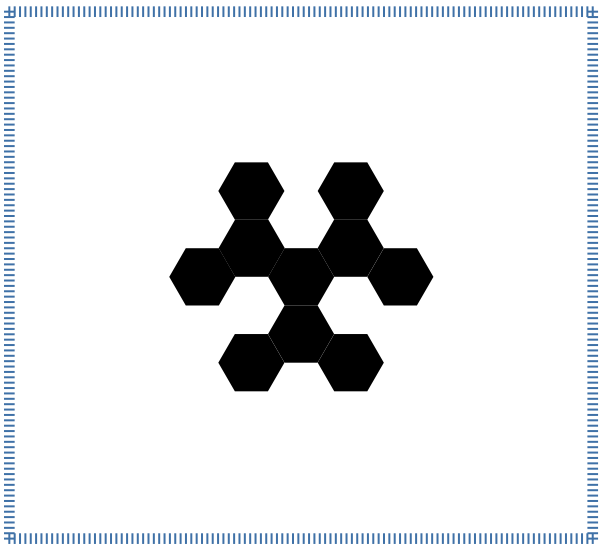


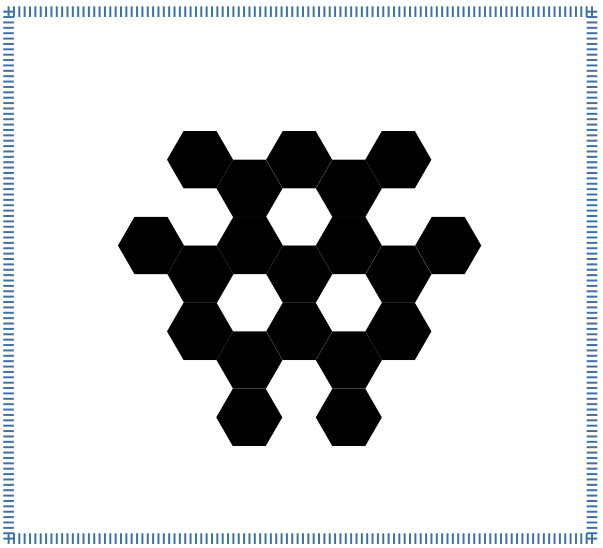


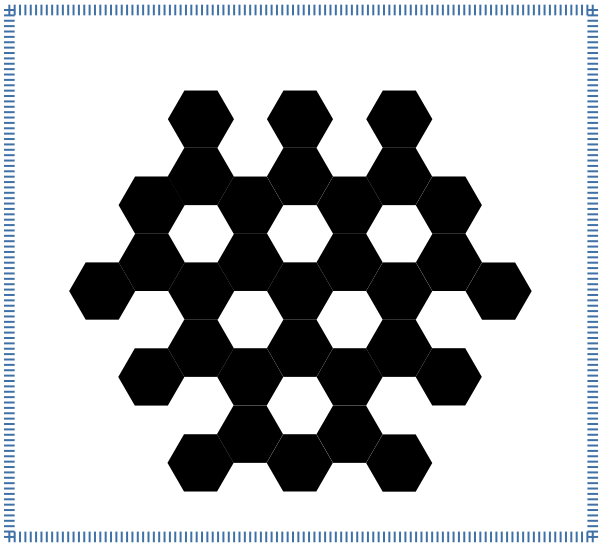


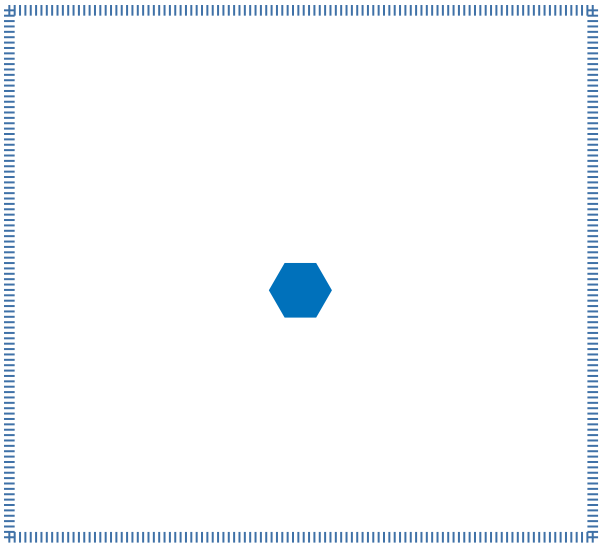


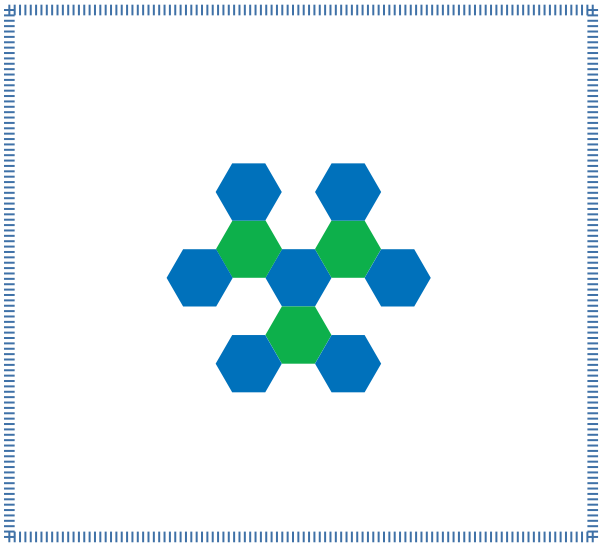


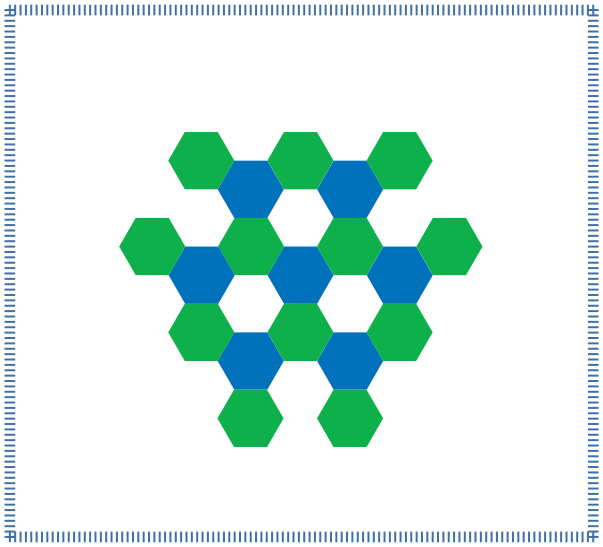


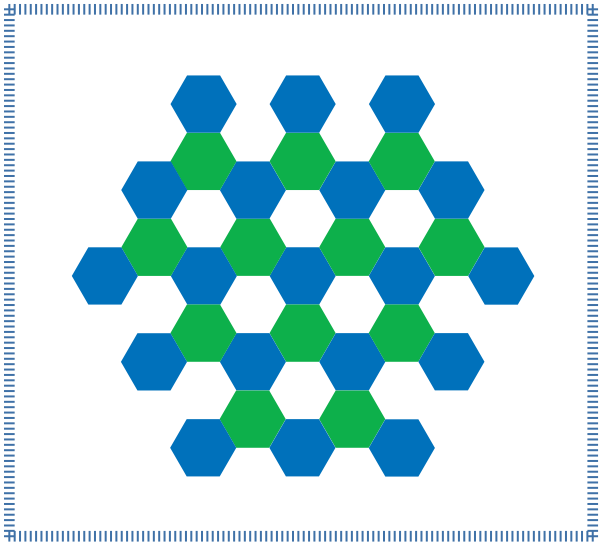












Reflect After Day 1

After the last activity of Day 1 Patterns Module has concluded, please reflect on student learning by responding to the following questions:

1. What novel ideas that students shared were unexpected to you?

2. For which students did the *Sort and Label* activity work well? How do you know?

3. What new strengths did you learn about your students through the *Classmate Bingo* activity?

TEACHER NOTES

The activities on Day 2 support students to investigate another aspect of patterns: how they change within sequences given in terms of images. By focusing on what is changing and what is not changing between terms of a sequence, students develop approaches for describing the iterative rule for generating subsequent terms of a sequence with the kinds of descriptions that are well-suited to finding closed forms or rules for an arbitrary or nth term of the sequence.

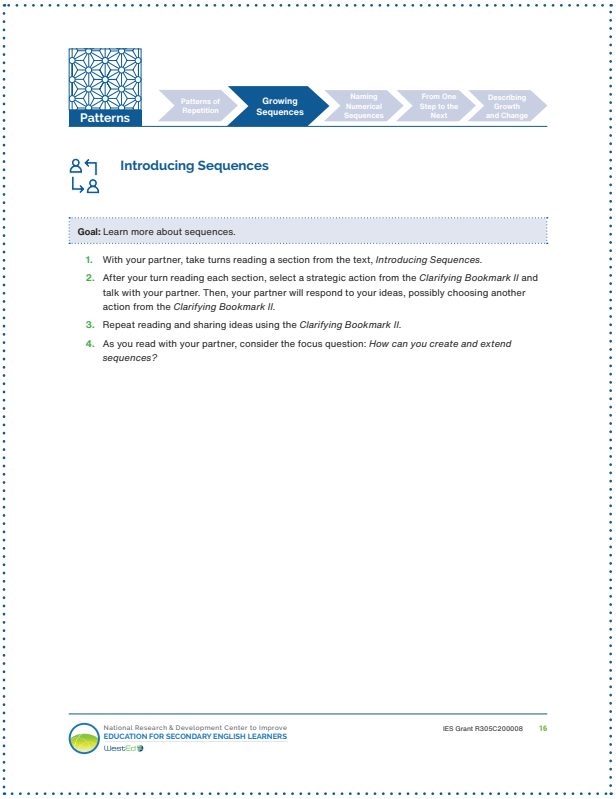
Introducing Sequences

Students will read a brief text about sequences and discuss it with a partner.

Purpose and Rationale

The reading *Introducing Sequences* presents a working definition of sequences and prompts students to consider how to create and extend sequences.

Engaging in this reading with a partner provides an opportunity for students to make connections from their own experiences to the technical definition of a sequence.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Introducing Sequences

Goal: Learn more about sequences.

1. With your partner, take turns reading a section from the text, *Introducing Sequences*.
2. After your turn reading each section, select a strategic action from the *Clarifying Bookmark II* and talk with your partner. Then, your partner will respond to your ideas, possibly choosing another action from the *Clarifying Bookmark II*.
3. Repeat reading and sharing ideas using the *Clarifying Bookmark II*.
4. As you read with your partner, consider the focus question: *How can you create and extend sequences?*

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Prepare

1. Read the *Introducing Sequences* text.
2. Consider what information students will use to answer the focus question or how they will answer it.

Enact and Observe

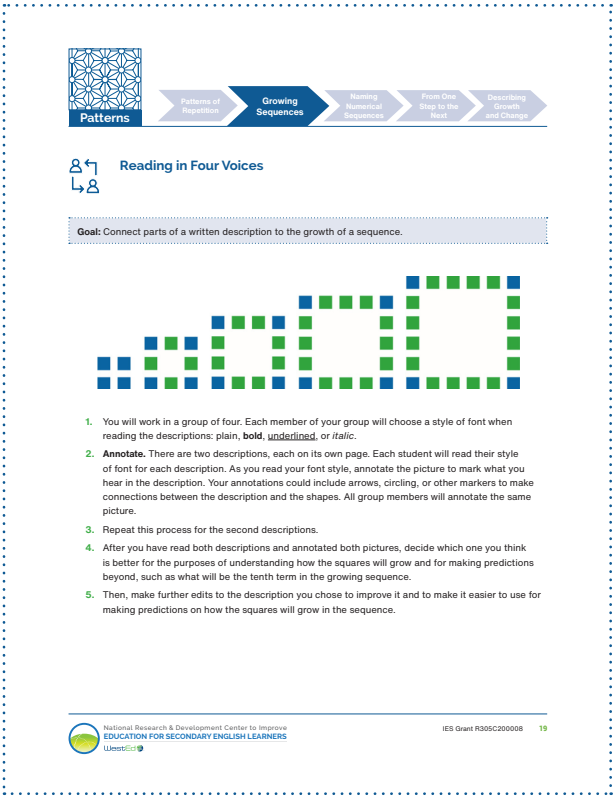
3. Invite student to take turns reading a section from the text, Introducing Sequences. Prompt students to select an action from the *Clarifying Bookmark II* and talk with their partner.
4. Encourage students to continue using the *Clarifying Bookmark II* in their discussion.

Close and Connect

5. Ask students to share any ideas with the full group about the focus question: *How can you create and extend sequences?*
6. Tell students that in the next task they will review descriptions of patterns to learn more about how to write about patterns.

Reading in Four Voices

The *Reading Four Voices* activity involves students reviewing four descriptions of a pattern and matching the elements of the description to a visual growing pattern.



Purpose and Rationale

The purpose of this activity is for students to learn more about how to describe growing patterns, and particularly how to refine the language of a description for clarity.

This activity provides students with sample language for describing a pattern. Through reviewing the sample descriptions and the patterns, students can refine the language.

Prepare

1. Students will work in groups of four.
2. Review the descriptions and annotate each description to show which part of the pattern the description matches.
3. Consider using a “fishbowl” discussion teaching strategy for setting up students for this activity. The fishbowl discussion teaching strategy includes an inner circle of students who will share out their thinking and an outer circle, where the rest of the class observes the activity and prepares to do the activity themselves. To use this strategy in the *Reading Four Voices* activity, identify students who can be in the inner circle while you are preparing to lead this activity.

Vignette that demonstrates the student learning opportunities from the "fishbowl" strategy for introducing the *Reading in Four Voices* activity

The teacher introduced the task by saying, "Annotate and make some marks, read the style of font for the description. That's what you'll mark on the picture. How do these words match our sequence there? Your annotations could include arrows, circles, or other markers."

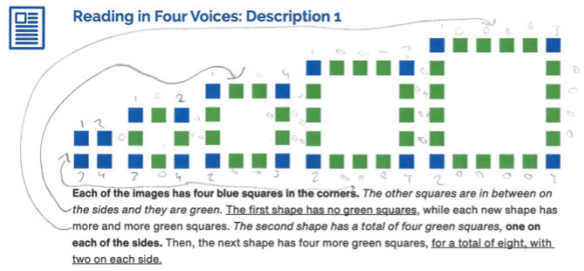
The fishbowl consisted of a graduate assistant, Melinda, the teacher, and Quincy. The order of reading followed the sequence in the text. The other five students were circled around the fishbowl of four people, who were seated. They were watching and annotating on their own papers as the four marked up theirs on the second read.

The teacher said, "We're going to draw...how the text is connected to the picture. See, I'm now connecting my text to the picture. You decide what's best for you."

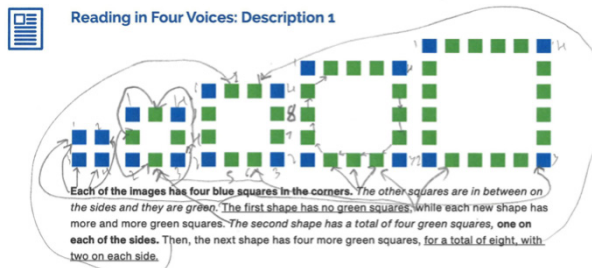
Osha and Xenia were taking notes, intently.

The class then moved into small groups. One small group consisted of Nasir (underline), Melinda (bold), Quincy (bold), and Rahim (plain). They annotated during the second reading, individually drawing arrows back and forth.

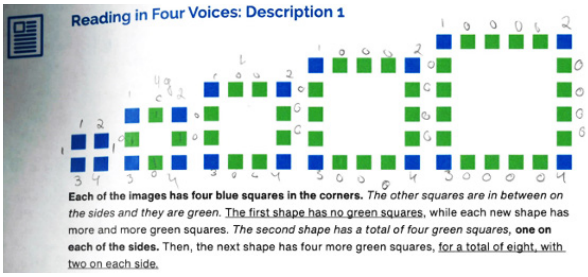
Nasir



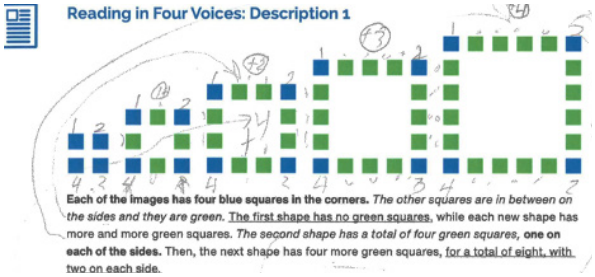
Quincy



Melinda



Rahim



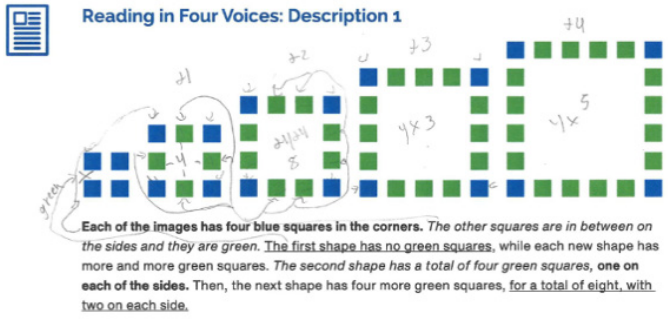
Enact and Observe

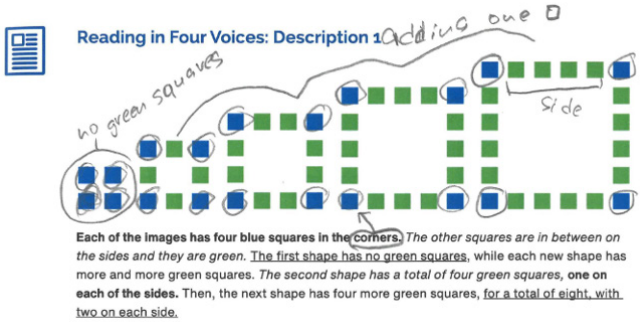
4. Distribute the *Reading in Four Voices* handout and the two handouts that show the growing patterns and the sample description below it. Tell students that each member of their group will choose a style of font when reading the descriptions—plain, **bold**, <u>underlined</u>, or *italic*—and ill annotate the picture to mark what they hear in the description.
5. After reading both descriptions, the students will choose which description they think is the best for understanding how the squares will grow and for making predictions beyond, such as what will be the tenth term in the growing sequence. Prompt students to be specific about why a specific description is best, and remind them that they can revise and refine the language in the description.

Close and Connect

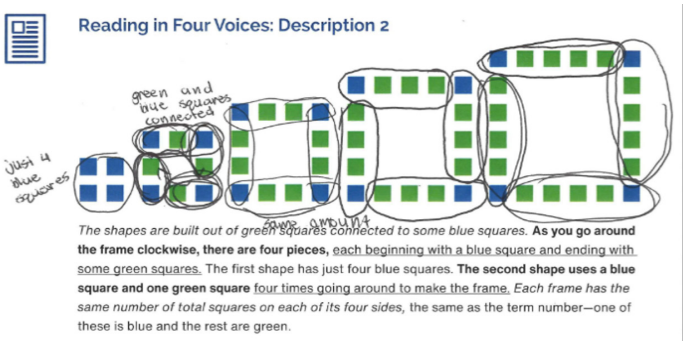
6. After all the students have chosen a description, close the activity by calling on different groups to share which description they selected and why. Set up the whole-class discussion to enable representations from each group to agree, disagree, or add new rationales.
7. Tell students that they will continue to explore sequences and work on describing them in the next task

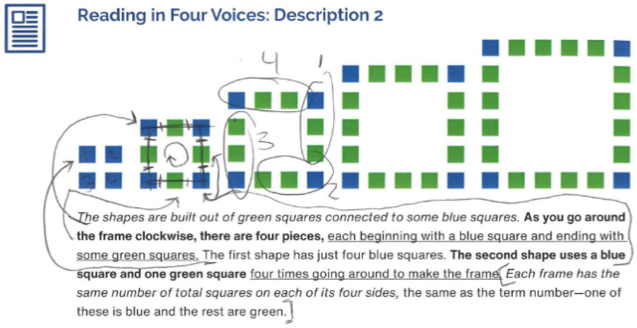
Student work closely aligned to the intention of the text



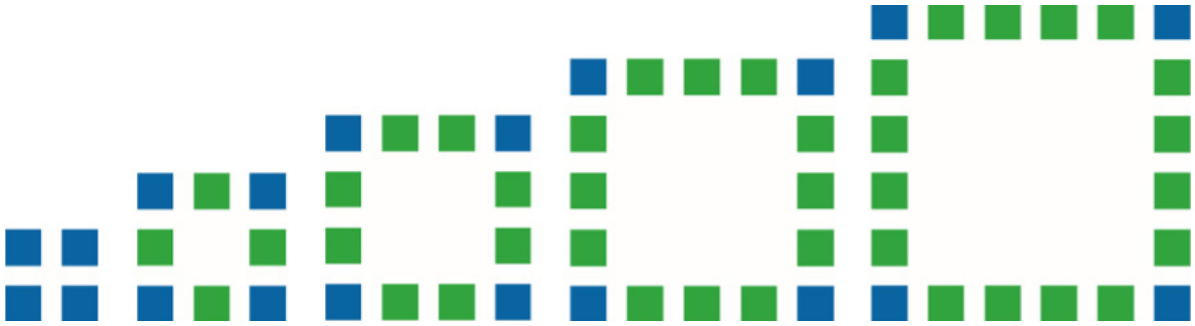


Student work closely aligned to the intention of the text



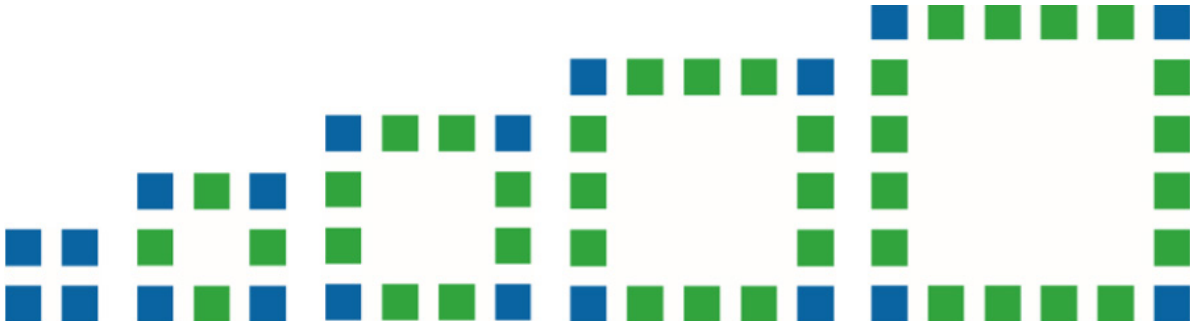


Reading in Four Voices: Description 1



Each of the images has four blue squares in the corners. *The other squares are in between on the sides and they are green.* The first shape has no green squares, while each new shape has more and more green squares. *The second shape has a total of four green squares,* **one on each of the sides.** Then, the next shape has four more green squares, for a total of eight, with two on each side.

Reading in Four Voices: Description 2



The shapes are built out of green squares connected to some blue squares. **As you go around the frame clockwise, there are four pieces,** each beginning with a blue square and ending with some green squares. The first shape has just four blue squares. **The second shape uses a blue square and one green square** four times going around to make the frame. *Each frame has the same number of total squares on each of its four sides,* the same as the term number—one of these is blue and the rest are green.

Expert Groups, Individual Matrix, and Expert Matrix

Students will work in small groups, exploring visual sequences and how they grow and change, and becoming experts on their visual sequence. Then, they will share their visual sequence and what they learned back with their base group.

Purpose and Rationale

The purpose of this task is to facilitate student learning about visual sequences, their key elements, and how best to describe them. In the *Expert Matrix*, students will detail what they learned about their visual sequence from their work in their expert group and what they learned from other members of their base group.

Through exploring visual sequences, students can deepen their understanding of how to describe a visual sequence and be prepared for learning about additional sequences from their peers. Independent reflection before group sharing provides students with an opportunity to collect and organize their ideas. Students then build their math knowledge and math identity when they share their learnings as experts back in their base group.

Prepare

1. You will organize students into base groups of three students. Each member of that group will be in a different expert group for the next set of activities. Then, the three members will come back to their base group for the final activity.
2. In each Expert Group, students will engage in the following activities: *Describe and Order*, *Structure and Regularity Guidance Card*, *Individual Matrix*, and *Expert Matrix*. Review the instructions for each activity.
3. Review each visual sequence and respond to each question in the *Expert Matrix*.

Enact and Observe

4. Distribute the materials for each visual sequence to each group. Each group only needs the materials for their expert group, specifically the *Expert Groups: Describe and Order* and the *Structure and Regularity Guidance Card*. Details on how to facilitate student learning for those activities follow.

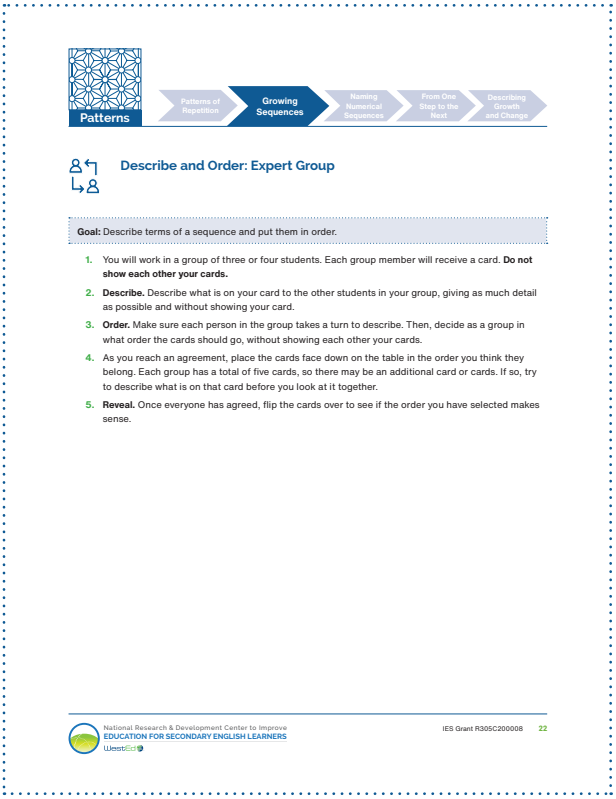
5. After students have completed the *Structure and Regularity Guidance Card*, distribute the *Individual Matrix* to each student while they are still in their expert groups. Students should first complete the focus questions in the *Individual Matrix* independently; then, they will share their responses and reach a consensus.
6. When students reach a consensus on each question in the *Individual Matrix*, distribute the *Expert Matrix* to them while they are still in their expert groups. Prompt students to write what they agreed on for each question in the column that applies to their expert group on the *Expert Matrix*. Students should have completed both the *Individual Matrix* and one column of the *Expert Matrix* while in their expert groups.
7. Explain to students that they will use the *Expert Matrix* to make notes on what they learn from the other groups, and then prompt students to go back to their base groups.
8. In base groups, students will take turns going question by question and sharing their responses to each question for their visual sequence.

Close and Connect

9. This is the last activity on Day 3. Prompt students to complete the Day 3 writing prompt.

Describe and Order: Expert Groups

The *Describe and Order* activity provides an opportunity for students to practice describing a term of a visual sequence with the goal of ordering the terms without viewing them first. Students work together to order the terms correctly. Then, they answer additional questions about their sequence in an individual matrix. Students will work in small groups, exploring their sequence and becoming experts in it. Then, they will share what they learned back with their base group.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Describe and Order: Expert Group

Goal: Describe terms of a sequence and put them in order.

1. You will work in a group of three or four students. Each group member will receive a card. **Do not show each other your cards.**
2. **Describe.** Describe what is on your card to the other students in your group, giving as much detail as possible and without showing your card.
3. **Order.** Make sure each person in the group takes a turn to describe. Then, decide as a group in what order the cards should go, without showing each other your cards.
4. As you reach an agreement, place the cards face down on the table in the order you think they belong. Each group has a total of five cards, so there may be an additional card or cards. If so, try to describe what is on that card before you look at it together.
5. **Reveal.** Once everyone has agreed, flip the cards over to see if the order you have selected makes sense.

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Purpose and Rationale

The purpose of this activity is for students to order terms in a sequence that is growing. They work in a group of four students to detail what they have learned about their sequence in their expert group and then share it with members of their base group.

Through describing their cards to others, students will see and say key elements and generate and exchange new ideas about what to look for when trying to order terms in a sequence.

They will also engage in detailing more sophisticated descriptions, seeking to make sure that the students in their group understand their term and place it correctly. Students will also learn more about describing sequences as they listen to how others describe the other terms in the sequence. Independent reflection before group sharing provides students with an opportunity to collect and organize their ideas. Students then build their math knowledge and math identity when they share their learnings as experts back in their base group.

Prepare

1. You will organize students into base groups of four students. Each member of that group will be in a different expert group for the *Describe and Order* and the *Individual Matrix*; then, the four members will come back to their base group when completing the *Expert Matrix.*
2. Review at least two sets of cards and describe the terms of them out loud to prepare for facilitating students as they describe and sequence the terms.
3. Create envelopes in which you have cut out up to five cards per sequence. While there are additional cards in this set up, this offers some flexibility in terms of grouping and an additional term for students to guess.

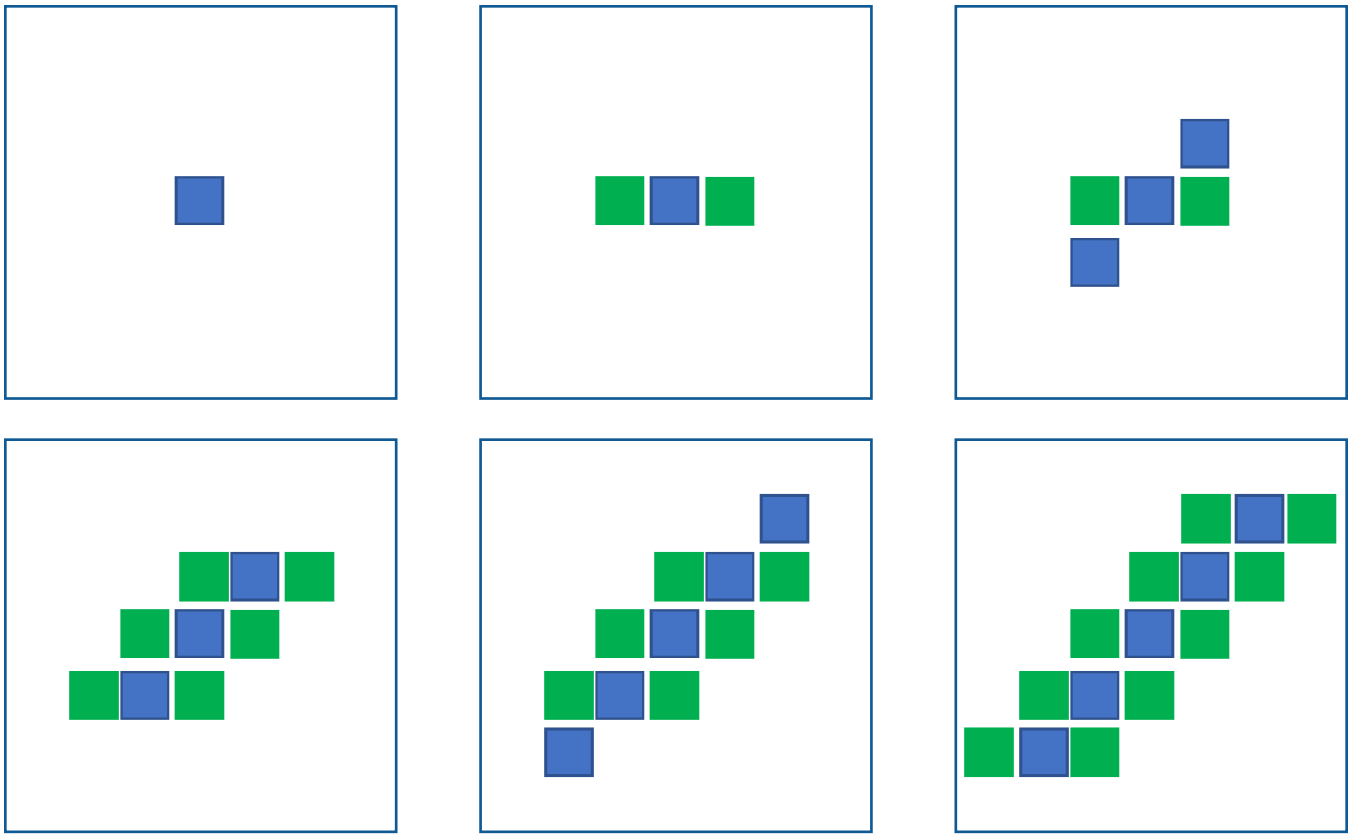
Enact and Observe

4. Distribute to each group an envelope with the *Describe and Order* cards and tell students that they will each take out one card that shows a term in a sequence.
5. Instruct students not to show their cards to their group members. Explain that they will describe their card to their group members, giving as much detail as possible and without showing the card. Students will then discuss how to order the cards without revealing them yet. As students agree on a sequence of the cards, they will place the cards face-down on the table. Once everyone has agreed on the order, students will flip the cards over to see if the order makes sense.
6. As students are working, make sure that they are providing enough detail as well as listening carefully to their peers. Encourage students to ask each other questions as they are describing and making connections from their cards to other cards.
7. After all groups have agreed on an order and revealed their cards, ask students to reflect on what kinds of information helped the most as they were trying to describe and order their sequence.

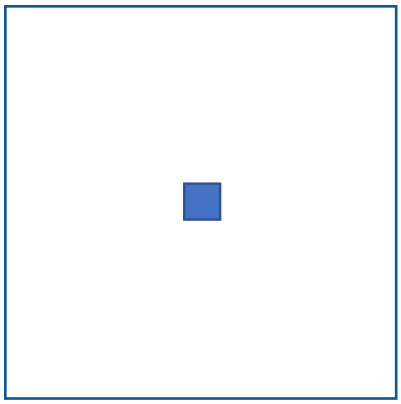
Close and Connect

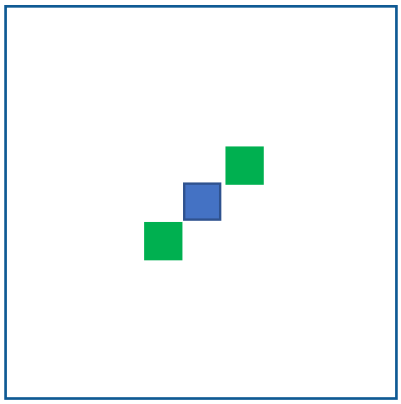
8. Explain to students that they will now with a partner on the *Structure and Regularity Guidance Card.*

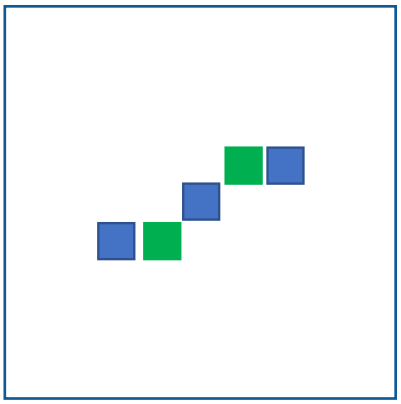
Describe and Order Cards: Pattern 1

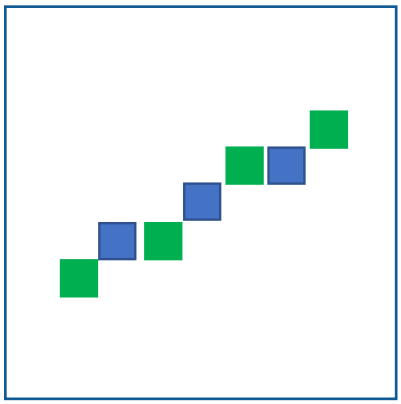


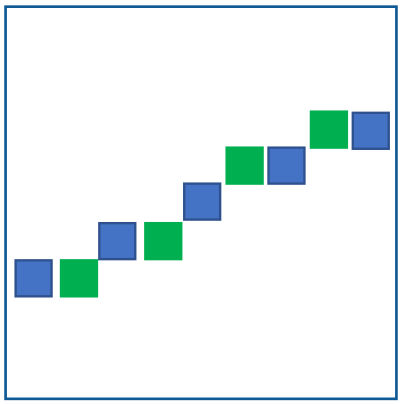
Describe and Order Cards: Pattern 2



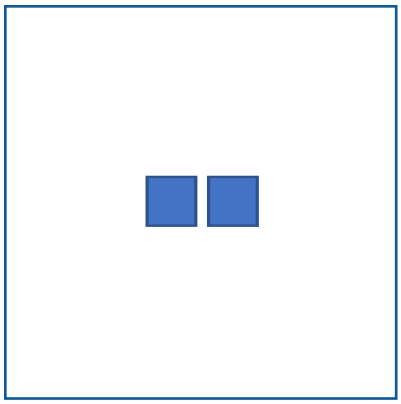


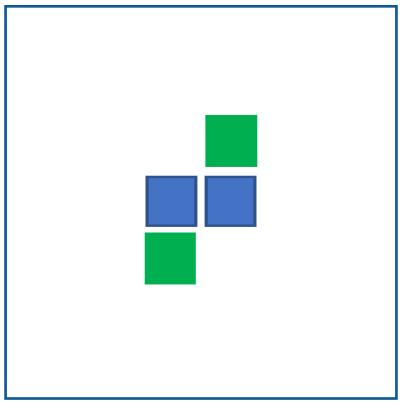


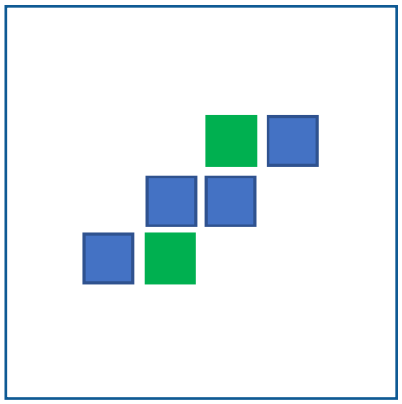


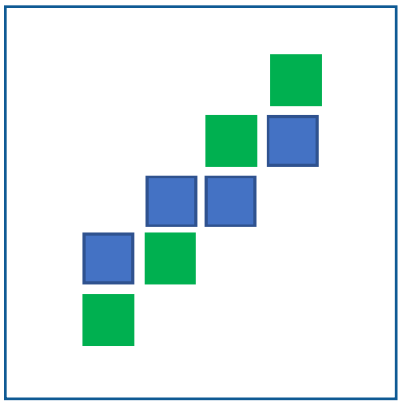


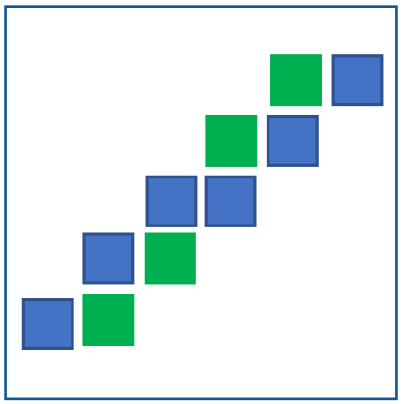
Describe and Order Cards: Pattern 3



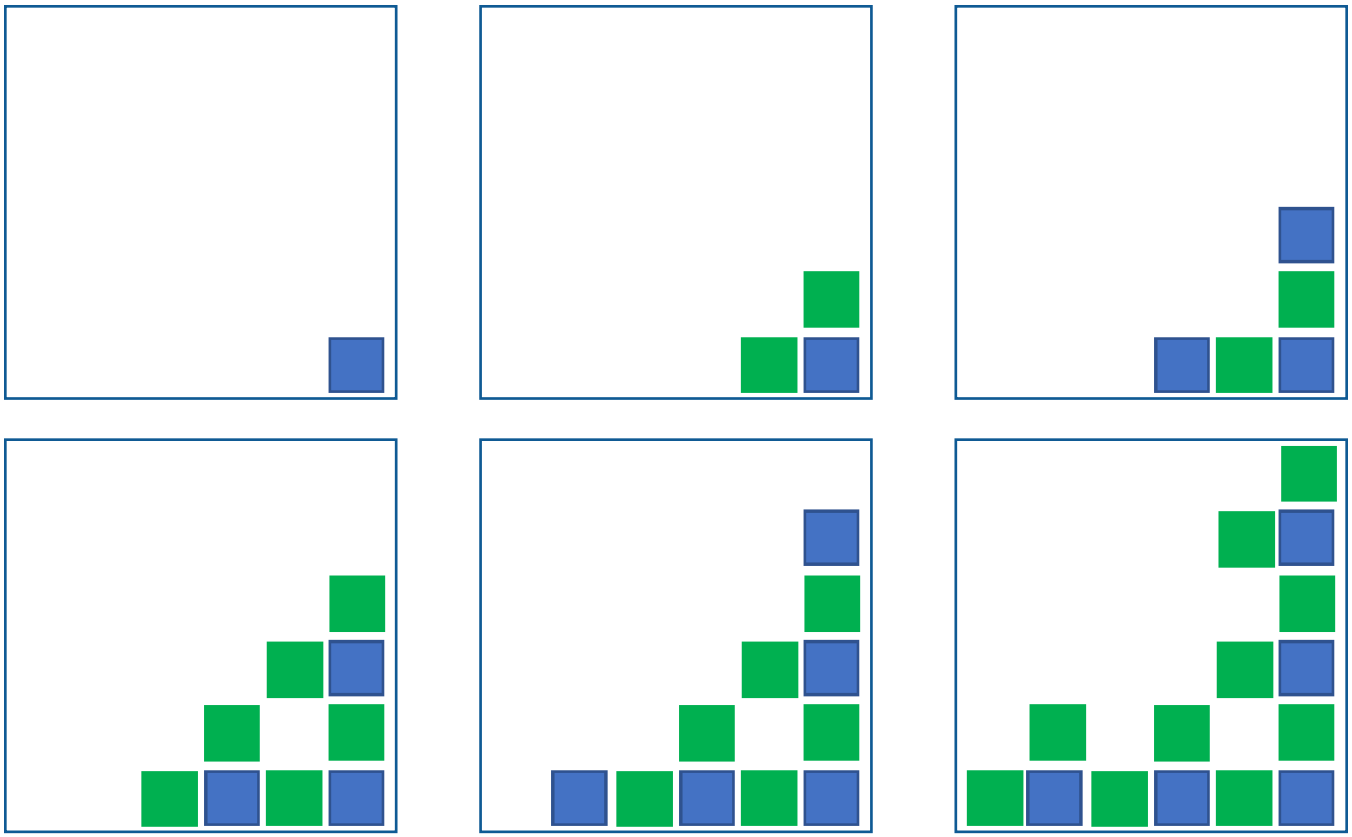




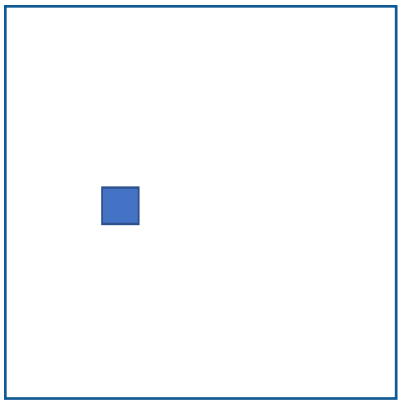


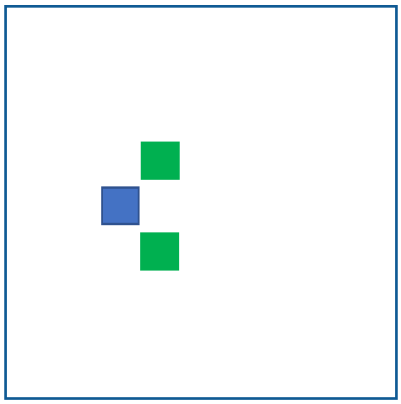


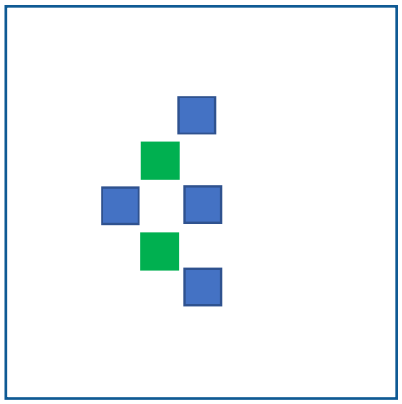
Describe and Order Cards: Pattern 4

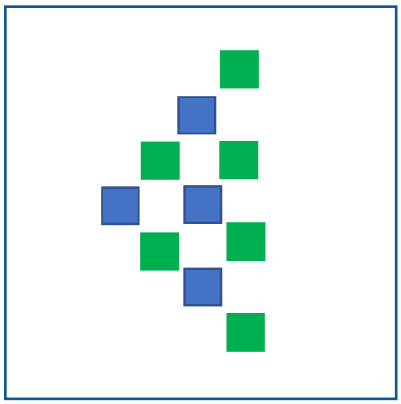


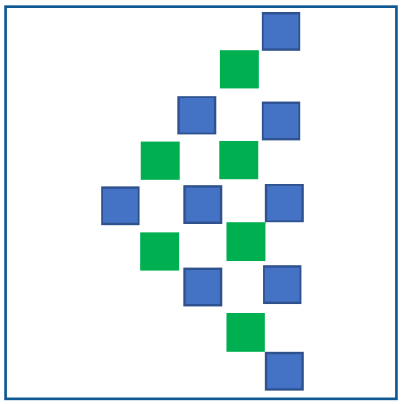
Describe and Order Cards: Pattern 5



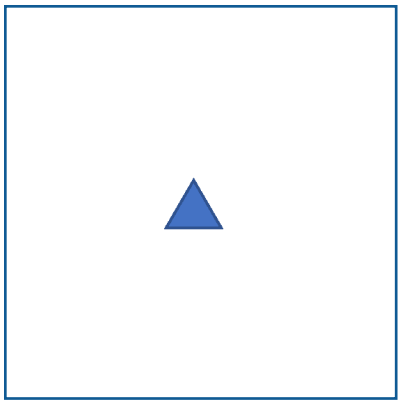


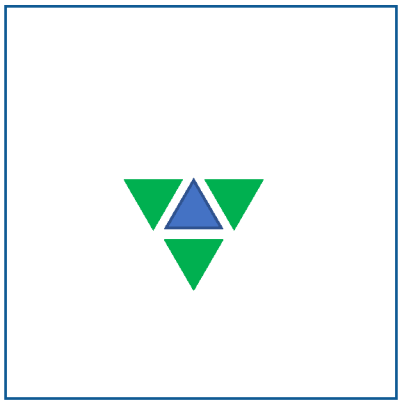






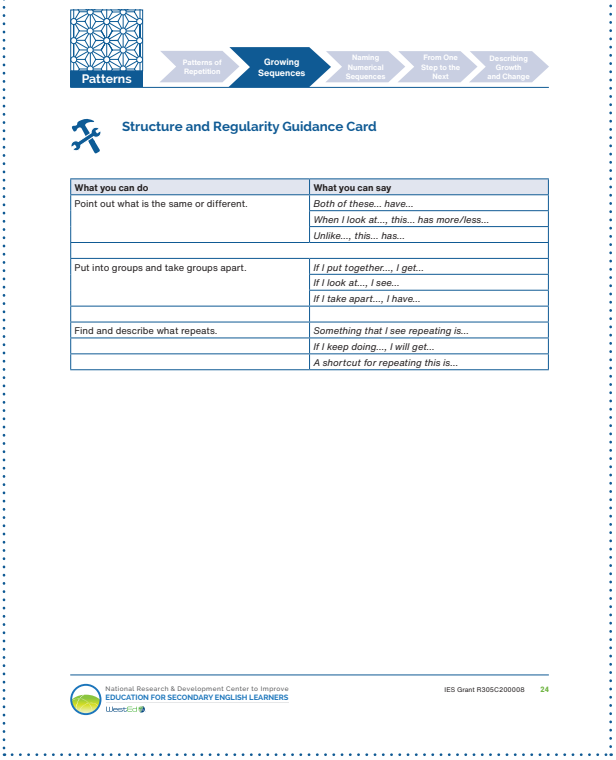
Describe and Order Cards: Pattern 6





Structure and Regularity Guidance Card: Expert Group

The *Structure and Regularity Guidance Card* serves to guide students in discussing key ideas of sequences, specifically similarities in terms and what repeats from one term to the next.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Structure and Regularity Guidance Card

What you can do	What you can say
Point out what is the same or different.	*Both of these... have...*
	When I look at..., this... has more/less...
	Unlike..., this... has...
Put into groups and take groups apart.	*If I put together..., I get...*
	If I look at..., I see...
	If I take apart..., I have...
Find and describe what repeats.	*Something that I see repeating is...*
	If I keep doing..., I will get...
	A shortcut for repeating this is...

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Purpose and Rationale

The purpose of this activity is for students to describe the terms of a sequence in ways that highlight structure and regularity.

The *Structure and Regularity Guidance Card* provides choices on what students can do and say when reviewing the sequence that they just revealed. Engaging in the actions offered by the card enables students to engage in more sophisticated mathematical discussion in later activities.

This activity facilitates students learning about description and how to detail it to others.

Prepare

1. Review the *Structure and Regularity Guidance Card* and think about how you will model it with students.

Enact and Observe

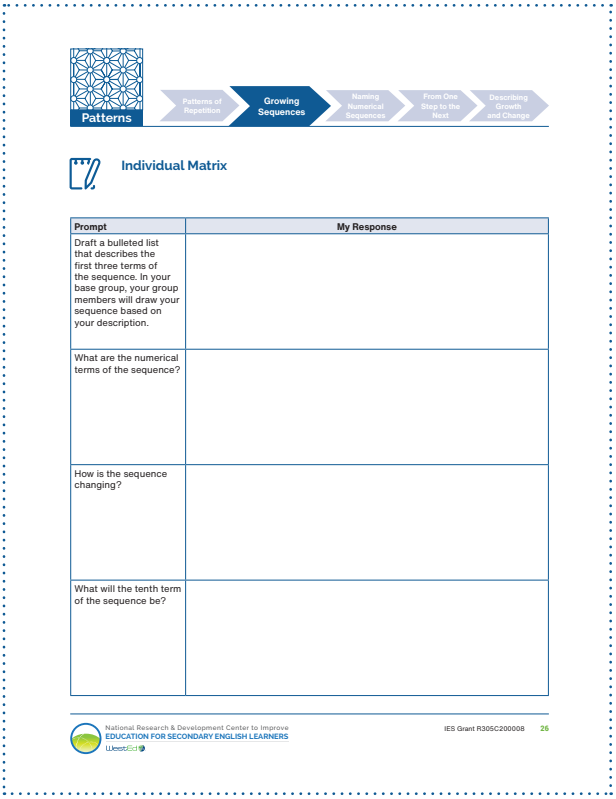
2. Have the expert groups of four split into two pairs. Distribute the *Structure and Regularity Guidance Card* handout.
3. Prompt students to take turns talking about the pattern from the last activity, using the sentence starters on the *Structure and Regularity Guidance Card.*

Close and Connect

4. After students have worked with their partner, all students in each expert group should transition to reviewing the questions on the *Individual Matrix.*

Individual and Expert Matrix: Expert Group

The *Individual Matrix* and *Expert Matrix* set up opportunities for students to connect visual sequences, numbers, and rules.



Purpose and Rationale

The purpose of this activity is for students to learn about a visual sequence and then deepen their understandings of visual sequences emphasized in the expert groups.

This activity facilitates students learning about description and provides an opportunity for students to deepen their understandings of additional sequences.

Prepare

1. Complete the *Expert Matrix* before students engage in the activity.

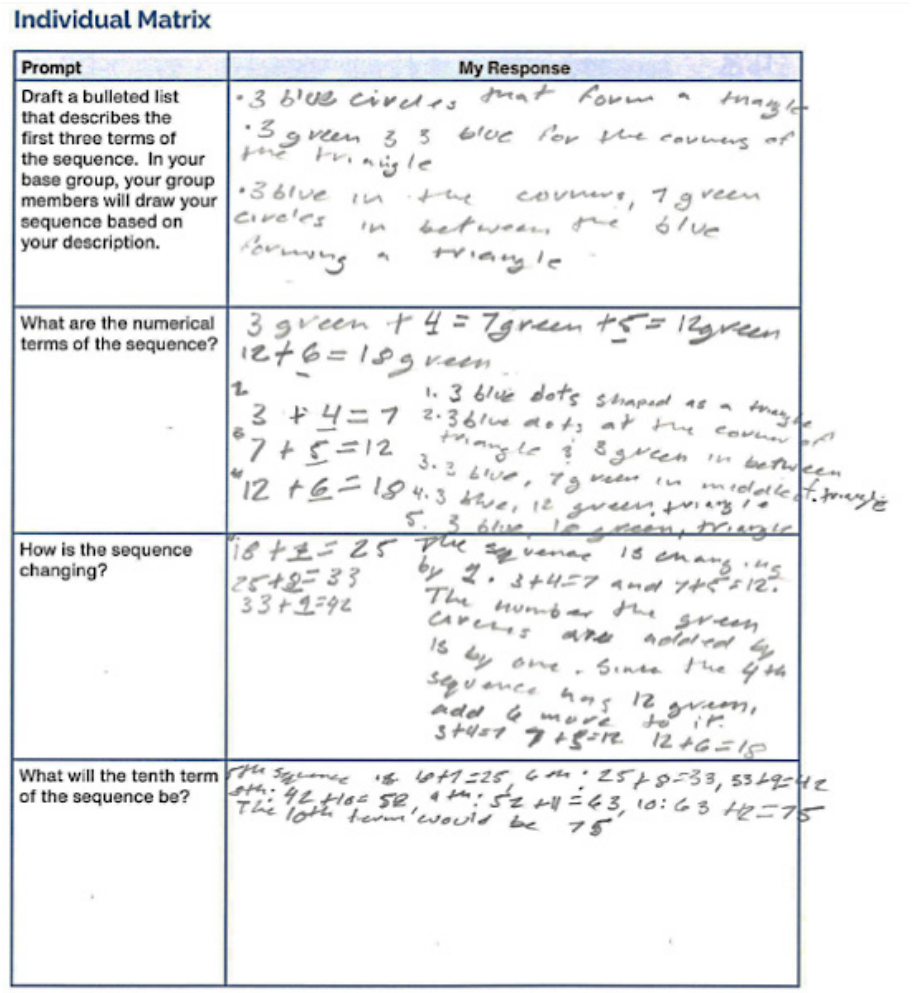
Enact and Observe

2. Distribute the *Individual Matrix* to each student while they are still in their expert groups.
   Students should complete the focus questions in the *Individual Matrix* independently before they engage in sharing their responses and reaching a consensus.
3. Encourage students to practice their description more than once as peers in their base groups will use that description to draw the visual sequence.
4. When students reach consensus, distribute the *Expert Matrix* to them while they are still in their expert groups. Prompt students to write what they agreed on for each question in the column that applies to their expert group on the *Expert Matrix*.
5. Explain to students that they will use the *Expert Matrix* to make notes on what they learn from the other groups when they are back in their base groups.

Close and Connect

6. Prompt students to return to their base groups and review each of the descriptions with their peers. In base groups, students will take turns going question by question and sharing their responses to each question for their visual sequence.
7. Completing the *Expert Matrix* is the last activity on Day 2. Thank students for sharing their description and working in their groups to become "experts" on a sequence. Share that the activities on Day 3 will continue to focus on sequences and how they grow. Prompt students to complete the *Daily Writing Prompt*.

Examples of students' individual matrices



Individual Matrix

Prompt	My Response
Draft a bulleted list that describes the first three terms of the sequence. In your base group, your group members will draw your sequence based on your description.	• 3 blue circles that form a triangle • 3 green 3 3 blue for the corners of the triangle • 3 blue in the corners, 7 green circles in between the blue forming a triangle
What are the numerical terms of the sequence?	3 green + 4 = 7 green + 5 = 12 green 12 + 6 = 18 green 2. 3 + 4 = 7 3. 7 + 5 = 12 4. 12 + 6 = 18 1. 3 blue dots shaped as a triangle 2. 3 blue dots at the corners of triangle & 3 green in between 3. 3 blue, 7 green in middle of triangle 4. 3 blue, 12 green, triangle 5. 3 blue, 18 green, triangle
How is the sequence changing?	18 + 7 = 25 25 + 8 = 33 33 + 9 = 42 The sequence is changing by 2. 3+4=7 and 7+5=12. The number the green corners are added by is by one. Since the 4th sequence has 12 green, add 6 more to it. 3+4=7 7+5=12 12+6=18
What will the tenth term of the sequence be?	The sequence is 6+7=25 4th: 25+8=33, 5th: 42+10=52, 6th: 52+11=63, 10: 63+12=75 The 10th term would be 75

Examples of students' individual matrices

Individual Matrix

Prompt	My Response
Draft a bulleted list that describes the first three terms of the sequence. In your base group, your group members will draw your sequence based on your description.	The first one is three blue circles creating a triangle. In the second there are now 3 green squares withe the three blue squares. It forms a triangle but all the three blue circles are still on the edges. The third one has seven green squares in the middle and the blue ones one on each edge forming the triangle
What are the numerical terms of the sequence?	The first one had 3 blue dots forming a triangle, the second one has a triangle formed by the three blue dots in the corners and three inside the triangle, the third one has a triangle formed by three blue dots in the corners and 7 inside the triangle. The fourth one is a triangle formed by the three blue dots one in each corner and 6 green dots inside the triangle. The last one is a triangle formed by a blue dot in each corner and 10 green dots inside
How is the sequence changing?	All of the sides are increasing by one and for the centers you can add by the number order, you have 0 circles in the center and as it increases by 1 you can add 2 for the center amount in the third card, then you can add 3 to 3 for the center amount. You can also start by adding 4 to 3 which is 7, then 5 to 7, then 12 to 6 and then use the last amount.
What will the tenth term of the sequence be?	1) 3 2) 3+4=7 3) 7+5=12 4) 12+6=18 5) 18+7=25 6) 25+8=33 7) 33+9=42 8) 42+10=52 9) 52+11=63 10) 63+12=75

Vignette that demonstrates the student talk when completing the Individual Matrix

One group wrote their individual descriptions of the first three terms of the sequence (see below for Jaleel's description). Then, they shared their descriptions verbally with others, reading from their papers:

Melinda: The first one is small and it just one. The second have three green, and one blue, and it's growing bigger. That every thing more sequence gets more green and blues each time.

Zora: The first one is zero ...one hexagon. The hexagons are going around. The third one is three more blues and hexagons and going around the same way as the second responses, and each hexagon is three hexagons per each green.

Jaleel: The sequence is changing in order. First there is one blue shape, one blue and three greens and then the third card three blues and three greens, so the sequence is determined based upon the number.

Xenia: The first image show a blue hexagon in the middle. Then the second image shows the blue hexagon in the middle, and three green hexagons connected to it ...the second ...and shows six more blue hexagons.

Jaleel shared this description:

Individual Matrix

Prompt	My Response
Draft a bulleted list that describes the first three terms of the sequence. In your base group, your group members will draw your sequence based on your description.	The Sequence Changes in order first there was 1 blue Shape in the second Card the 1 blue and 3 Greens and then the third 3 Greens and 7 blues so the Sequence is determined

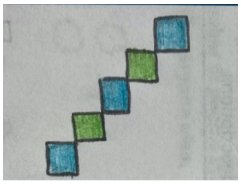
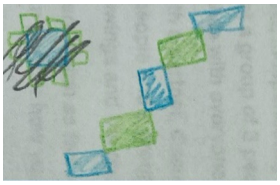
on how the

Vignette that demonstrates student sharing of descriptions when they return to base groups

After students compared their notes in their expert groups, they returned to their base groups. Hien, Melinda, and Quincy reunited to give their descriptions and see how effective they were for their peers. They took turns giving the following descriptions as the other students recorded their work, respectively.

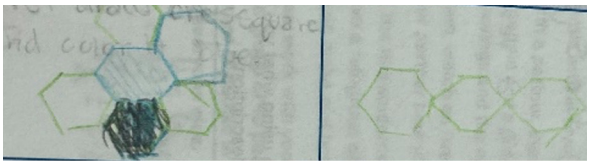
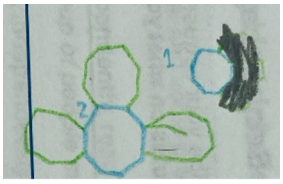
Hien: First draw one square and color it blue. Then draw two green squares of the corner. The corner of the one blue square diagonally. Then repeat the blue with the second step.

What Melinda and Quincy drew in response to Hien's description.

Melinda	**Quincy**
	

Melinda: First draw one small hexagon in blue. Second have three greens at the corners. [inhales] It's three sides and in each side there are hexagons

Hien drew three hexagons in a train, while Quincy drew hexagons with the approximate orientation described by Melinda.

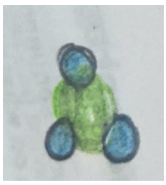
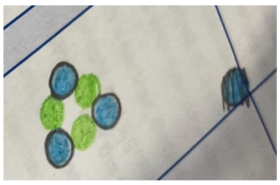
Hein	**Quincy**
	

Quincy: It's a triangle, and on the corner of every triangle, there is a blue circle. So, imagine a triangle, and on the corner draw a blue circle, but don't draw the triangle.

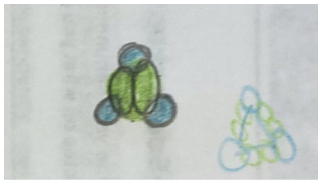
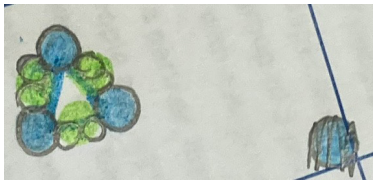
Quincy: So that's the first one, the second one, in between every blue there is one green...

Hien: One green circle?

Hein and Melinda drew two drawings to represent what Quincy shared.

Hein	**Melinda**
	

Quincy: Like that. You see how you got one green, instead of that, it's two.

Hein	**Melinda**
	

Reflect After Day 2

After the last activity of Day 2 Patterns Module has concluded, please reflect on student learning by responding to the following questions:

1. What did you notice about students' descriptions in their *Individual Matrix* or how they described their cards in the *Describe and Order* activity?

2. For which students did the *Structure and Regularly Guidance* card work well? How do you know?

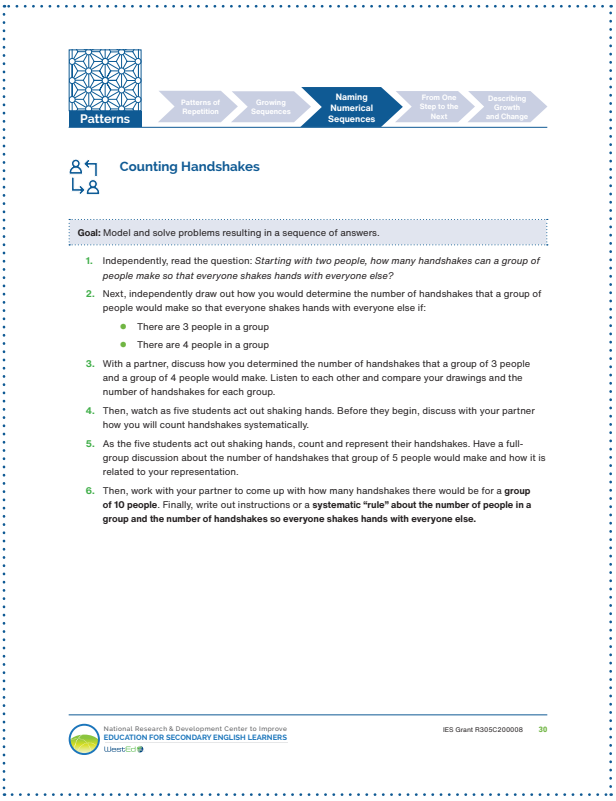
3. In what ways did the *Expert Matrix* support student identify as a mathematician?

TEACHER NOTES

The main purpose of Day 3 is to provide an opportunity for students to explore numerical sequences in real-world situation. Students will also learn notation and conventions for writing numerical sequences as lists of numbers.

Counting Handshakes

This activity involves students in modeling a sequence, specifically determining how many handshakes are possible if everyone in the group shakes hands once with one other person.



Purpose and Rationale

The purpose of this task is to engage students in exploring sequences and identifying how the number of handshakes possible changes with the number of people.

This task offers students an opportunity to act out and draw the possible number of handshakes, working from a group of three or four people to a group of 10 people or more shaking hands.

Prepare

1. Review the task and solve it independently using a visual representation. Be ready to explain your visual representation to others.
2. Students will work independently and with a partner. Determine how you will group students.
3. A small group of five students will act out shaking hands. Determine how you will choose five students to act out shaking hands.

Enact and Observe

4. Distribute the *Counting Handshakes* handout. Read the questions aloud, and then give students three minutes to work independently to draw out how they would determine the number of handshakes that a group of 3 people and a group of 4 people would make so that everyone shakes hands with everyone else.
5. After students work on the task independently, ask them to discuss it with their partner. If students have not solved the task independently first, they can still work with a partner and discuss the number of handshakes a group of 3 people and a group of 4 people would make.

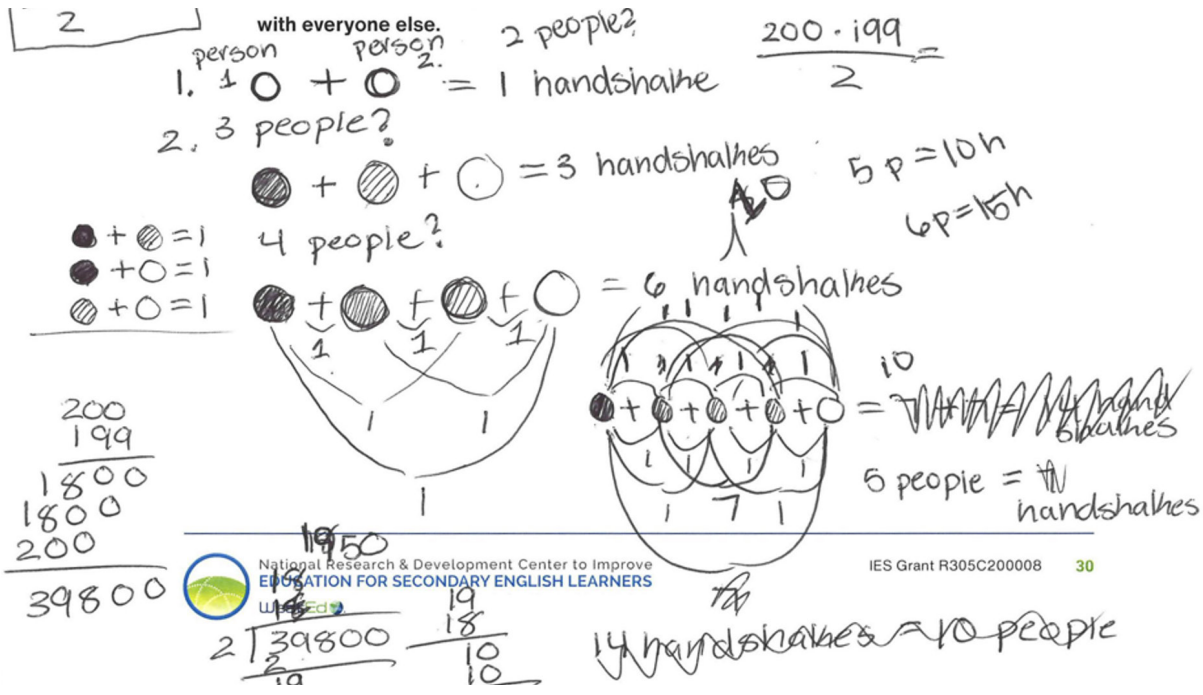
6. Then, invite five students to act out shaking hands. Before they begin, ask each group of students to determine how they are going to count the handshakes that are made.
7. After the five students act out shaking hands, have a full group discussion about the number of handshakes that five people would make.
8. Students will then work with a partner to come up with how many handshakes a group of 15 people would make and with a "rule" about the number of people in a group and the number of handshakes, so everyone shakes hands with everyone else.

Close and Connect

9. Have two small groups share the rule that they wrote for the number of people in a group and the number of handshakes, so everyone shakes hands with everyone else. Note the similarities and differences between the rules written by the two groups.
10. Share with students that the next activity will introduce specific notation for writing a rule for a sequence, and then students will have an opportunity to co-create and name their own numerical sequences.

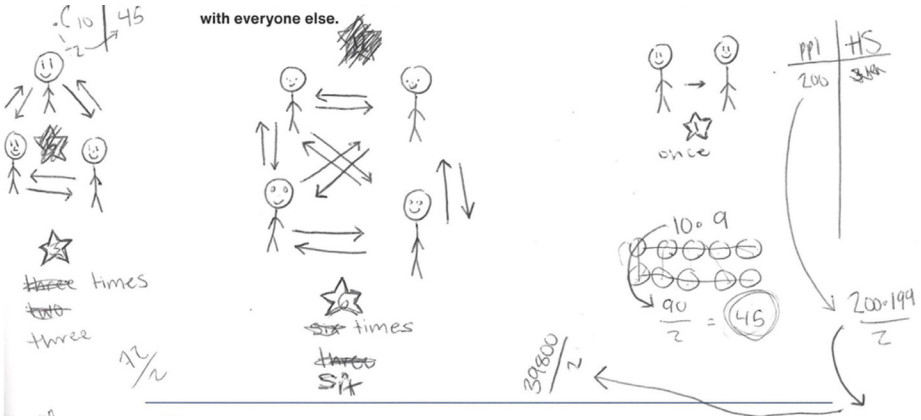
Student work on *Counting Handshakes* problem

Brenda's graphical representation has the potential to make connections to triangular numbers.



Student work on *Counting Handshakes* problem

Lola's drawing shows two connections between two people, demonstrated by pairs of arrows in opposite directions; this element of the drawing has the potential to lead students to recognize when they are "double counting" and to help them develop a systematic approach.

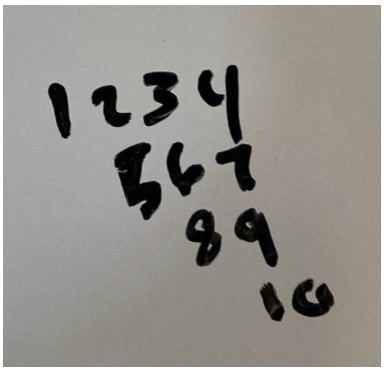


Review what Osha recorded on the board.

What could you do next to help students develop a formula from what they did?

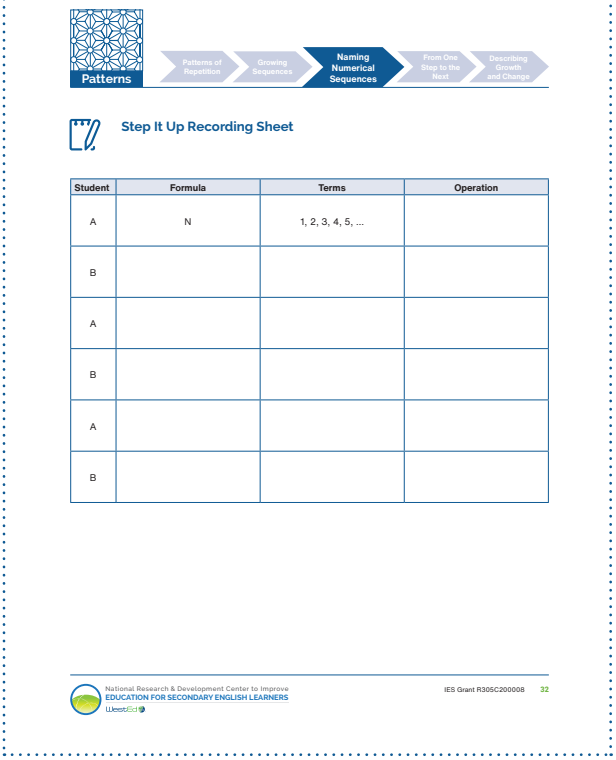
Vignette that demonstrates the importance of student representations of the *Counting Handshakes* problem and teacher noticing

Five students—Melinda, Nasir, Rahim, Hien, and Uzma—stand up. They count in different ways as Osha records. First, they shake hands "down the line" (i.e., Melinda shakes hands with everyone then steps off). Osha records this as a triangle. The other approach is to skip handshakes, which Osha records on the board as a circular diagram with dots and arrows showing handshakes.



Naming Sequences and Step it Up

After a brief introduction to sequences, students co-create and name numerical sequences, exploring arithmetic operations that can be applied to every term of a sequence and how such operations change or do not change the relationships between successive terms of a sequence.



Purpose and Rationale

The slides on naming sequences present sequence notation and terms. *In Step it Up*, students learn how to create and operate on sequences, reviewing the terms and operations that led to them.

By creating sequences themselves, students can explore the process as well as the final product and can investigate how to arrive at numerical sequences in more than one way.

Prepare

1. Determine how you will group students for this activity.
2. Review the slides on sequence notation. Also review the activity and take note of how students could end up with the same sequence through a different order of operations (for example, adding two and then multiplying by two would give the same numerical sequence as multiplying by two and then adding four).

Enact and Observe

3. Present the slides on *Naming Sequences*. Ask students: *How does the sequence 2N relate to N? What questions do you have about how to use the sequence notation?*
4. Distribute the *Step It Up* handout. Read the directions aloud to all students. Then, with one student, model the first two turns of suggesting an operation, writing the formula, and listing the terms. Ask students to follow along, but not to copy down this sample.

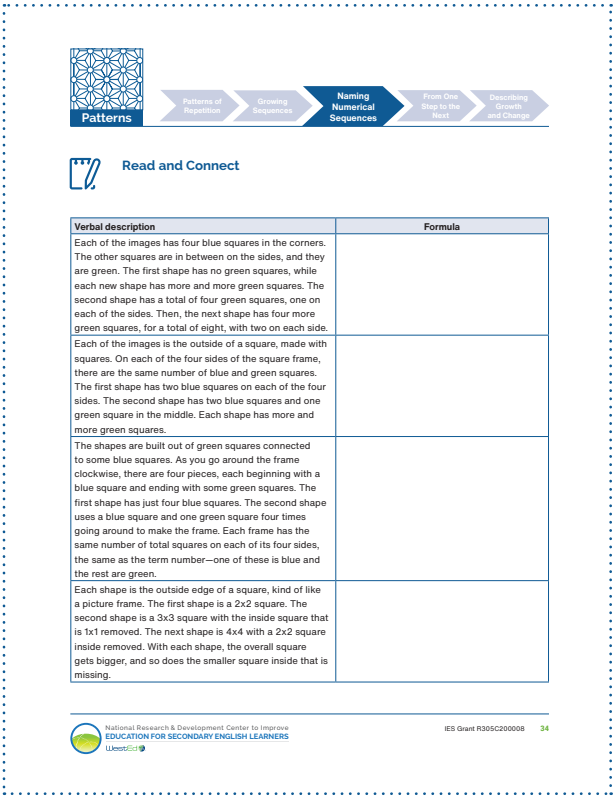
5. As students begin to work with a partner, encourage them to share their computations out loud with each other.
6. When they have completed the chart, students should then discuss with their partner if there is another way to arrive at the same numerical sequence or other operations that would yield the same numerical sequence. Encourage students to be ready to justify how another formula would arrive at the same numerical sequence.

Close and Connect

7. Review some of the examples that students have generated and select two resulting sequences that have the same common difference. Write these sequences on the board and have students talk about the patterns that they see.
8. Prompt students to review their sequences and share if they think there is another way they could have used different operations to get the same ending sequence.
9. Tell students that in the next task they will develop a tool that focuses on the differences between terms in the sequence to use to better understand and classify numerical sequences.

Read and Connect: Formulas

Matching verbal descriptions to formulas can support students' understandings of the elements needed in a verbal description and how to represent those elements in a formula.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Read and Connect

Verbal description	Formula
Each of the images has four blue squares in the corners. The other squares are in between on the sides, and they are green. The first shape has no green squares, while each new shape has more and more green squares. The second shape has a total of four green squares, one on each of the sides. Then, the next shape has four more green squares, for a total of eight, with two on each side.	
Each of the images is the outside of a square, made with squares. On each of the four sides of the square frame, there are the same number of blue and green squares. The first shape has two blue squares on each of the four sides. The second shape has two blue squares and one green square in the middle. Each shape has more and more green squares.	
The shapes are built out of green squares connected to some blue squares. As you go around the frame clockwise, there are four pieces, each beginning with a blue square and ending with some green squares. The first shape has just four blue squares. The second shape uses a blue square and one green square four times going around to make the frame. Each frame has the same number of total squares on each of its four sides, the same as the term number—one of these is blue and the rest are green.	
Each shape is the outside edge of a square, kind of like a picture frame. The first shape is a 2x2 square. The second shape is a 3x3 square with the inside square that is 1x1 removed. The next shape is 4x4 with a 2x2 square inside removed. With each shape, the overall square gets bigger, and so does the smaller square inside that is missing.	

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Purpose and Rationale

This task engages students in carefully reading descriptions and connecting elements of a verbal description to a formula.

When students attend to the structure of descriptions, including their parts and numbers within those parts, they are better able to make connections with algebraic formulas that provide a "closed form" to compute an arbitrary term of the sequence.

Prepare

1. Students will work in groups of four. Determine how you will group students for this activity.
2. Review each of the verbal descriptions and the formulas. Which elements of the descriptions may be challenging for students? Which formulas will students be most familiar with already?

Enact and Observe

3. Distribute the *Read and Connect* handout. Explain to students that they will each receive a formula and will examine it and identify the parts in it. Students will take turns listening to their group members read the descriptions, and then they will discuss if they think that their formula will fit a description.
4. Encourage students to connect parts of the description to parts of the formula. Students should review the descriptions and formulas after they have matched them to make sure that they all agree with the matches.

5. Then, prompt students to construct the formulas from the descriptions. Encourage them to look at the numerical terms and note differences in the sequences.

Close and Connect

6. Close the activity by first asking two student groups to share how they matched the descriptions and formulas. Prompt students to share which parts of the description matched which part of the formulas.
7. Close this activity by asking students to share how they constructed the formulas and other noticings they made about each sequence. Encourage students to describe any strategies they used when matching sequences.
8. Prompt students to complete the *Daily Writing Prompt.* Tell students that they will continue talking about patterns the next day, specifically solving real-world problems.

Read and Connect: Formulas Cards

$4+4(N-1)$	$4(N+1)-4$
$4N$	$(N+1)^2-(N-1)^2$

Reflect After Day 3

After the last activity of Day 3 Patterns Module has concluded, please reflect on student learning by responding to the following questions:

1. How were students' representations of the *Handshake Problem* similar or different? In what ways, if any, did students use the representations they drew to determine how many handshakes a group of 15 people would make?

2. What unique strategy did students use when connecting formulas to descriptions?

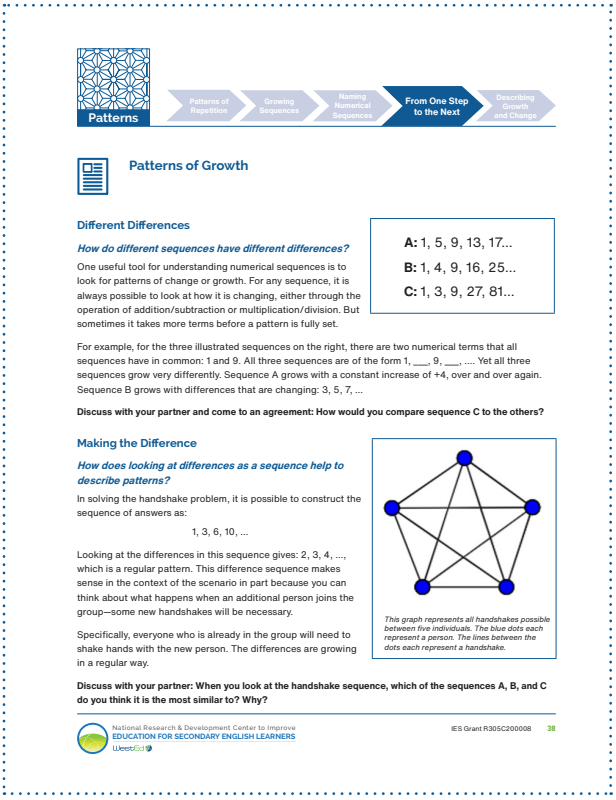
3. What language practices did students engage in during the *Step it Up* activity?

TEACHER NOTES

On Day 4, students will solve problems set in real-world contexts that will require them to identify the recursive patterns that generate subsequent terms of a sequence.

Patterns of Growth

Differences in sequences can help to explain the type of sequence and the type of growth. Through focusing on differences in sequences, students can view arithmetic, geometric, and quadratic patterns of growth through the nature of the changes, rather than focusing on a step-rule. Students can use the differences to help describe patterns and find rules that help identify subsequent terms.


Patterns
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Patterns of Growth

Different Differences

How do different sequences have different differences?

One useful tool for understanding numerical sequences is to look for patterns of change or growth. For any sequence, it is always possible to look at how it is changing, either through the operation of addition/subtraction or multiplication/division. But sometimes it takes more terms before a pattern is fully set.

A: 1, 5, 9, 13, 17...
B: 1, 4, 9, 16, 25...
C: 1, 3, 9, 27, 81...

For example, for the three illustrated sequences on the right, there are two numerical terms that all sequences have in common: 1 and 9. All three sequences are of the form 1, ___, 9, ___, ... Yet all three sequences grow very differently. Sequence A grows with a constant increase of +4, over and over again. Sequence B grows with differences that are changing: 3, 5, 7, ...

Discuss with your partner and come to an agreement: How would you compare sequence C to the others?

Making the Difference

How does looking at differences as a sequence help to describe patterns?

In solving the handshake problem, it is possible to construct the sequence of answers as:

1, 3, 6, 10, ...

Looking at the differences in this sequence gives: 2, 3, 4, ..., which is a regular pattern. This difference sequence makes sense in the context of the scenario in part because you can think about what happens when an additional person joins the group—some new handshakes will be necessary.

Specifically, everyone who is already in the group will need to shake hands with the new person. The differences are growing in a regular way.

This graph represents all handshakes possible between five individuals. The blue dots each represent a person. The lines between the dots each represent a handshake.

Discuss with your partner: When you look at the handshake sequence, which of the sequences A, B, and C do you think it is the most similar to? Why?

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Purpose and Rationale

The purpose of this task is for students to deepen their knowledge of sequences and differences.

This activity provides students with information on how different sequences have different differences, how differences can help describe patterns, and how other operations can be used to describe the growth and change of sequences. Through taking turns and working with a partner, students discuss mathematical text. The *Clarifying Bookmark II* provides students with ideas on what to say and how to apply related concepts.

Prepare

1. Make sure to decide how you will group students for this activity.
2. Read the text about sequences and differences. As you read the text, jot down some ideas about where you think students will be clarifying for each other through discussion.
3. Review and solve the two tasks in Part 2. Try to solve each task in a different way. Which elements of the *Bookmark to Understand the Problem* will be most helpful to students? Why?

Enact and Observe

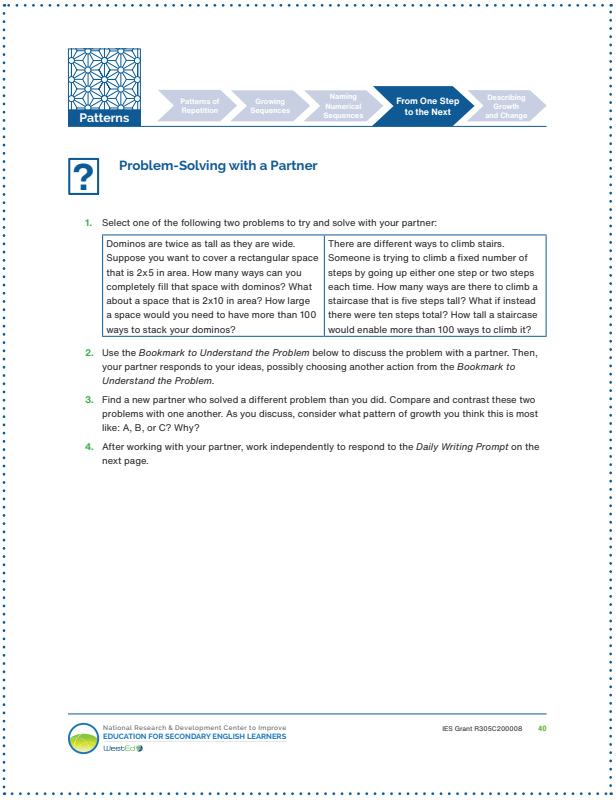
4. Organize students into groups of two and distribute a copy of *Patterns of Growth* and the *Clarifying Bookmark II* to each student. Review the directions. Tell students that they will each take a turn reading a section of the text and will also answer the questions embedded in the text.
5. After the first student reads, the student will choose a strategy from the *Clarifying Bookmark II* and use one of the formulaic expressions to share ideas; then, the second student will also choose a formulaic expression from the *Clarifying Bookmark II* to share ideas. Students will alternate reading and continue taking turns until they have finished reading.

Close and Connect

6. Bring students together and first ask them to share with their partner one new idea that they learned from the reading. Encourage students to share with the whole class what their partner said.
7. Tell students that we will now move to the problem-solving tasks on the *Problem Solving with a Partner* activity.

Problem Solving with a Partner

Problem solving with a partner can foster students' understanding of sequences and the strategies that they can use to explore them.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Problem-Solving with a Partner

1. Select one of the following two problems to try and solve with your partner:

Dominos are twice as tall as they are wide. Suppose you want to cover a rectangular space that is 2x5 in area. How many ways can you completely fill that space with dominos? What about a space that is 2x10 in area? How large a space would you need to have more than 100 ways to stack your dominos?	There are different ways to climb stairs. Someone is trying to climb a fixed number of steps by going up either one step or two steps each time. How many ways are there to climb a staircase that is five steps tall? What if instead there were ten steps total? How tall a staircase would enable more than 100 ways to climb it?

2. Use the *Bookmark to Understand the Problem* below to discuss the problem with a partner. Then, your partner responds to your ideas, possibly choosing another action from the *Bookmark to Understand the Problem*.
3. Find a new partner who solved a different problem than you did. Compare and contrast these two problems with one another. As you discuss, consider what pattern of growth you think this is most like: A, B, or C? Why?
4. After working with your partner, work independently to respond to the *Daily Writing Prompt* on the next page.

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Purpose and Rationale

The purpose of this task is for students to deepen their knowledge of sequences and differences through applications and discussions with a partner.

Students will engage in problem solving and discussion with their partner. The *Bookmark to Understand the Problem* can facilitate their problem solving and specifically their work with identifying what the problem is asking and how to represent the problem.

Prepare

1. Students will work with the partner that they read the *Patterns of Growth* reading with.
2. Complete both of the tasks. Identify different strategies students may use to solve the tasks and annotate the tasks to prepare to use them in this activity.

Enact and Observe

3. Distribute the *Problem Solving with a Partner* handout. Students will read both tasks and select one to work on with their partner.
4. Prompt students to use the *Bookmark to Understand the Problem* to discuss the task. Students should take turns choosing an action from the *Bookmark to Understand the Problem* to guide their discussion. Encourage students to do at least two actions as they solve the task. Students should show enough of their problem-solving approach so they can explain their work to another student in the next step of this activity.

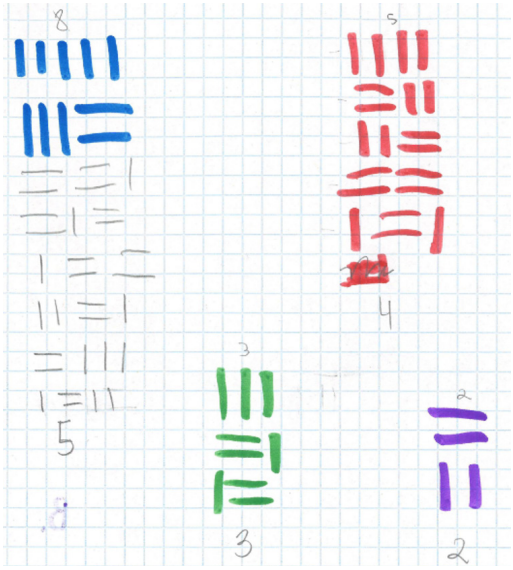
5. After solving one problem, students should find a new partner who solved the other problem, and compare and contrast the problems and their solutions. Encourage students to use the *Bookmark to Understand the Problem* as needed to guide their discussions with their new partner.
6. As students are working, prepare to choose two student pairs to present their problem-solving approach. You may choose one student pair for each problem or two pairs of students that solved the same problem but in a different manner. Make notes on what similarities and differences you would like students to notice about their peers' problem-solving approaches and why.

Close and Connect

7. Lead a discussion on the students' solved problems. Have one student pair present and share how they solved the task. Ask other students to consider what pattern of growth is represented in the problem (A, B, or C from the reading) and describe why. Highlight key strengths in students' representations or problem-solving approaches. Repeat the process for the sharing by a second student pair, and then also prompt students to consider the differences in approaches.
8. As this is the last activity on Day 4, prompt students to complete the *Daily Writing Prompt.* Share with students that we will close this module by working on patterns on Day 5.

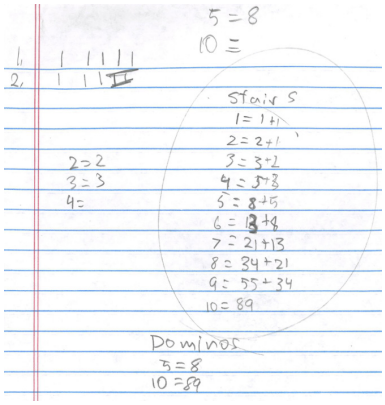
Student work on *Dominos Problem*

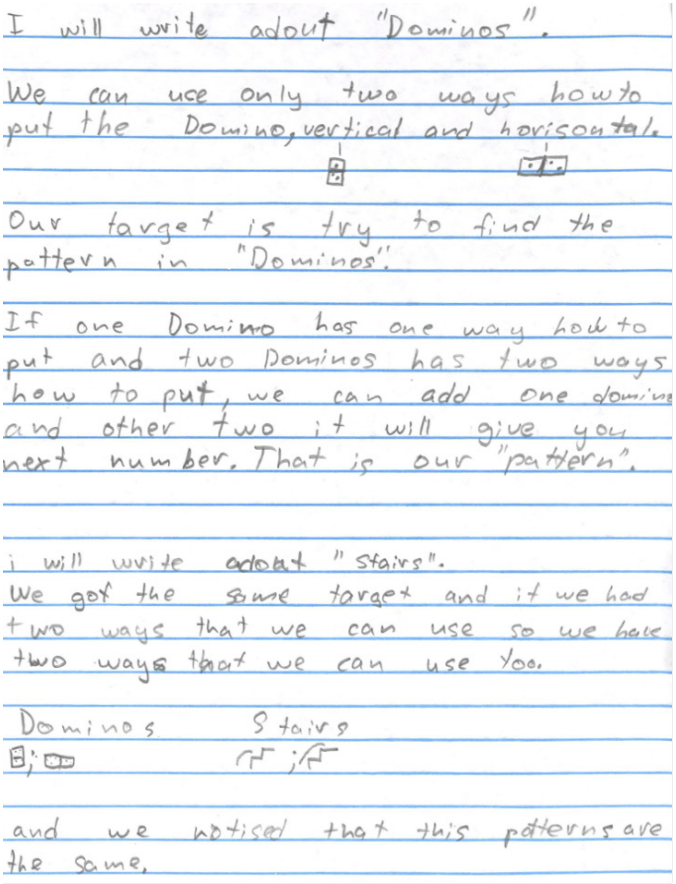
Melinda drew horizontal and vertical lines to represent how to arrange the dominos, drawing all possible 2x5 area arrangements of five dominos as well as similar sets of two dominos, three dominos, and four dominos.



Student work on *Dominos and Climbing Stairs Problem*

Boris shared a list to represent the number of ways to climb a staircase. When completing the Writing Extension Activity, he shared how to find the pattern for both the *Dominos and Climbing Stairs* problems.

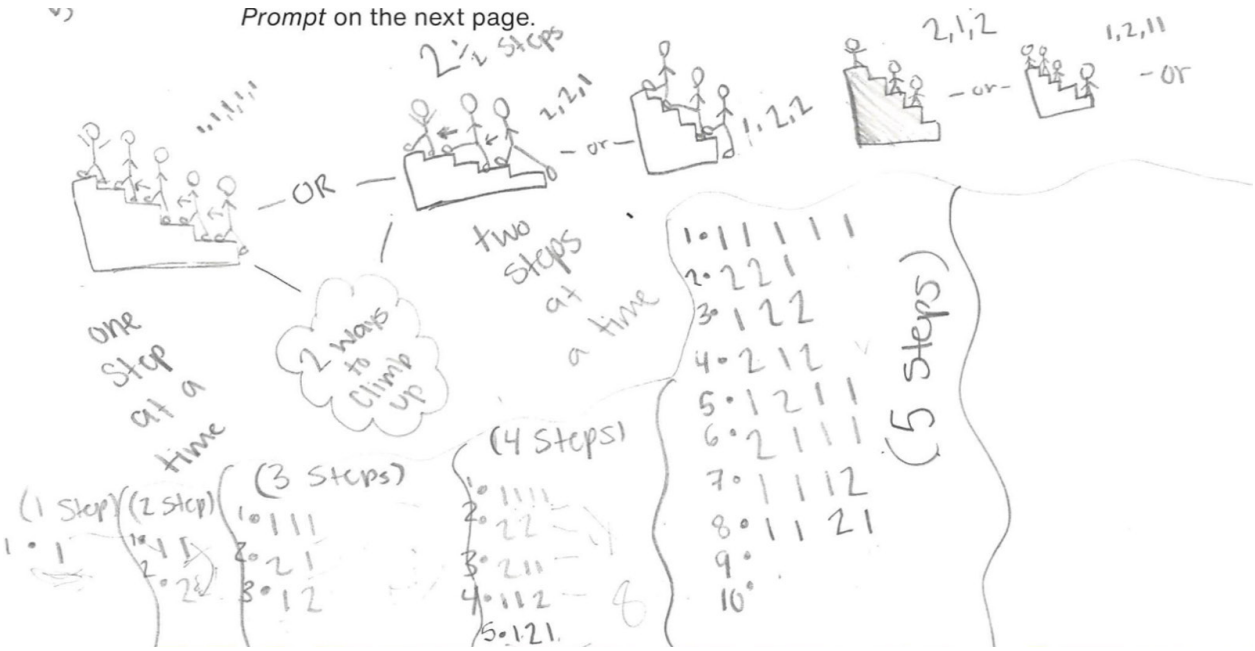


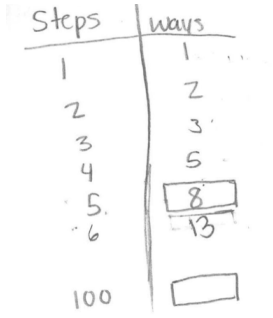


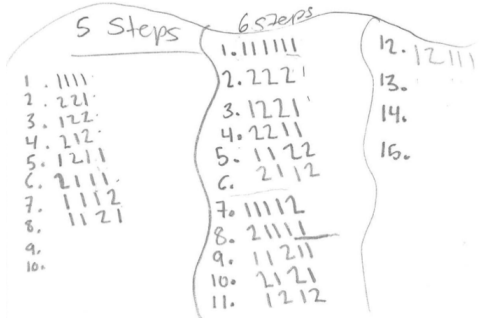
Student work on *Climbing Stairs Problem*

On the *Climbing Stairs* problem, Lola drew out the moves on the first few steps, then transitioned to an organized list, and then created a table.

Prompt on the next page.







Reflect After Day 4

After the last activity of Day 4 Patterns Module has concluded, please reflect on student learning by responding to the following questions:

1. What surprised you about students learning from the *Patterns of Growth* reading?

2. For which students did the *Bookmark to Understand the Problem* work well for their understanding of the problem? How do you know?

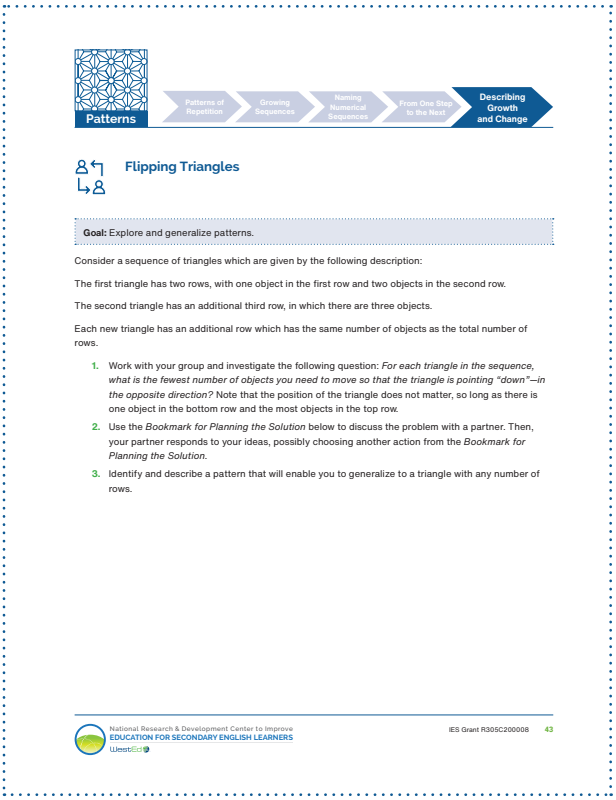
3. In what ways did working with a partner facilitate students' problem solving on the *Problem Solving with a Partner* tasks? Or, in what ways did working with a second partner on the second task facilitate student learning or problem solving?

TEACHER NOTES

Day 5 includes opportunities for students to continue to explore patterns as well as to reflect on their learning about patterns across the module.

Flipping Triangles

Two-dimensional patterns provide another representation of patterns and one that students can consider manipulating as they explore and generalize.



Purpose and Rationale

The purpose of this activity is for students to explore the given sequences of triangles as a two-dimensional pattern and then move on to identify a general pattern that can be used to describe a triangle with any number of rows. Students will use the *Bookmark to Planning a Solution* as they work.

This activity prompts students to consider how patterns grow and how to generalize a pattern.

Prepare

1. Review the *Flipping Triangles* task. Consider if students will have access to manipulatives, such as chips or blocks, or large poster paper or white board space to work together on this task.
2. Solve the *Flipping Triangles* task. What challenges do you anticipate students may have as they solve this task or that you may need to address? Which elements of the *Bookmark for Planning* for a Solution do you think will be most helpful as they work on this task?

Enact and Observe

3. Distribute the *Flipping Triangles* task and the *Bookmark for Planning for a Solution*. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered.
4. Students should record their solutions and their approach and be prepared to share them with others.

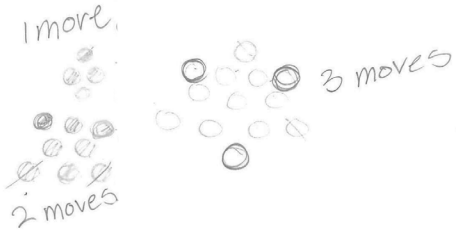
Close and Connect

5. Ask students to share their patterns by posting them on for other groups to see. Ask students to circle the room and review the work of other groups. Have them notice: *What is similar or different about the approach that other students took? What questions do you have for your classmates about their problem-solving approach?*
6. Then, close with a discussion of questions that students had for their classmates. Discuss any differences in patterns or approaches before moving on to the next task.

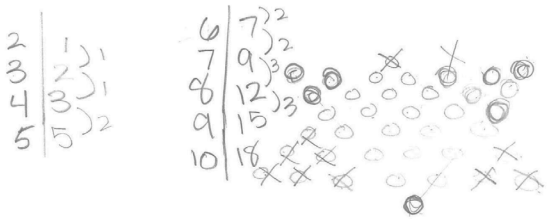
Student work

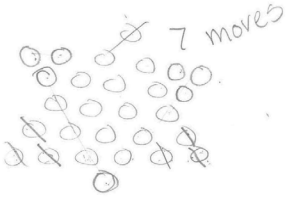
Brenda noted moves, tracked changes, and made a table and sequences.

1) Brenda noted moves for the first few changes

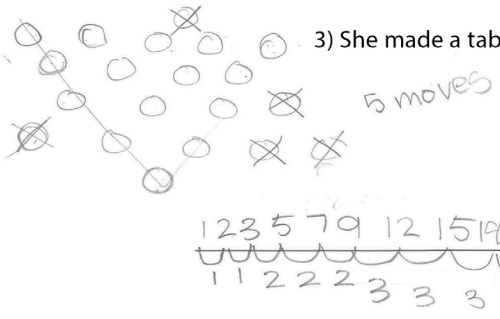


2) She tracked larger changes





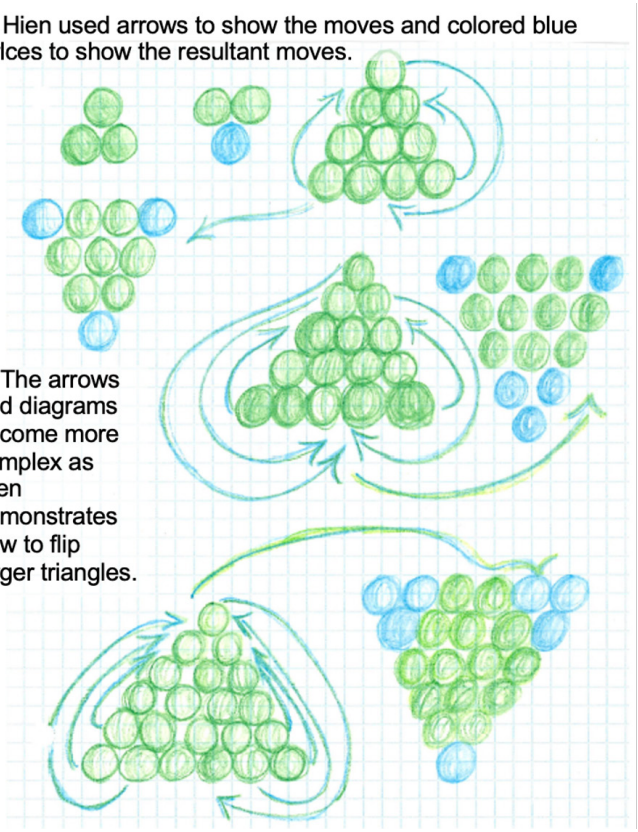
3) She made a table and sequence, identifying changes



Student work

Hien demonstrated how to invert the triangle by moving select cases.

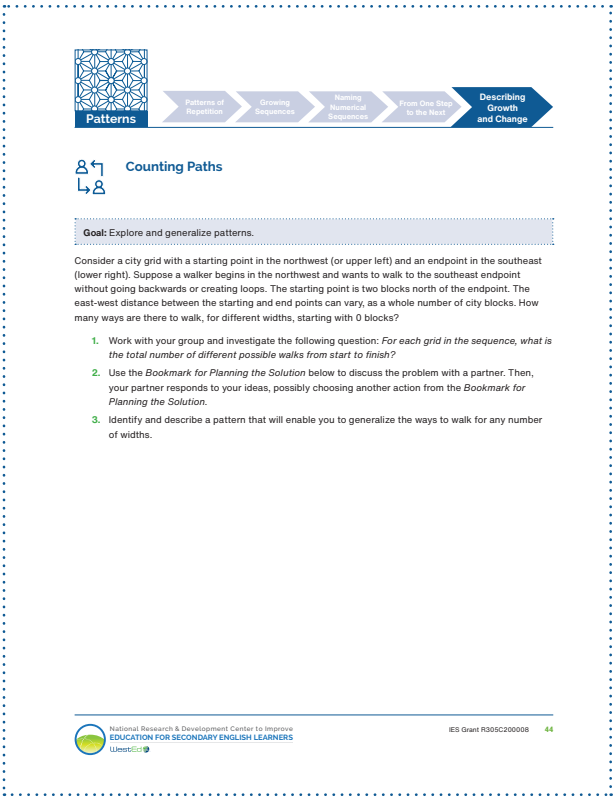
1) Hien used arrows to show the moves and colored blue cirlces to show the resultant moves.



2) The arrows and diagrams become more complex as Hien demonstrates how to flip larger triangles.

Counting Paths

Tasks that encourage students to identify a quantity through a rule rather than by counting can set up students to look for generalizable patterns.



Purpose and Rationale

The purpose of this activity is for students to explore the possible paths a walker can take and then describe a generalizable pattern for the ways to walk for any number of widths. This activity prompts students to identify a generalizable rule after they explore possible paths and as the number of possible paths seems too large to count. Students will use the *Bookmark to Planning a Solution* as they work. This activity prompts students to consider the generalizable pattern in number of ways to create a path from one point to another.

Prepare

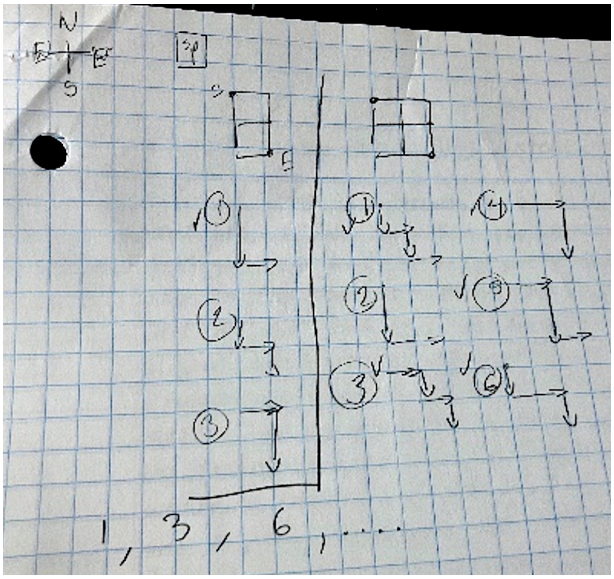
1. Solve the *Counting Paths* task. What challenges do you anticipate students may have as they solve this task or challenges that you may need to address? Which elements of the *Bookmark for Planning for a Solution* do you think will be most helpful as they work on this task?

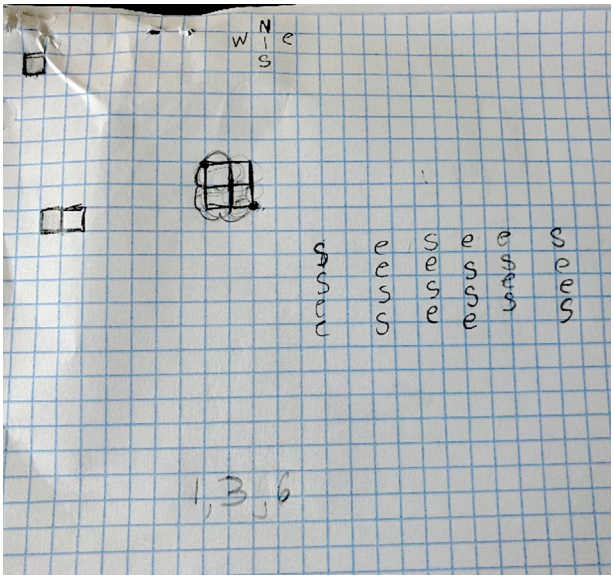
Enact and Observe

2. Distribute the *Counting Paths* task and the *Bookmark for Planning for a Solution*. As students work in their group, encourage them to listen to each other and repeat others' ideas as necessary to make sure that all ideas are considered.
3. Students should identify the number of walks to walk for different widths and record them, looking for a pattern. As students work, encourage them to review their sequences and look for generalizable patterns.
4. Ask students to record their solutions and their approach and be prepared to share them with others

Samples of Student Work

Students tend to take two broad approaches to systematically listing the different possibilities. The first is to draw the different paths, as shown on the left. The other is to list the moves (South or East), often by using an abbreviation. Either of these approaches can lend insight into why the overall sequence is growing the way that it is in terms of new paths.



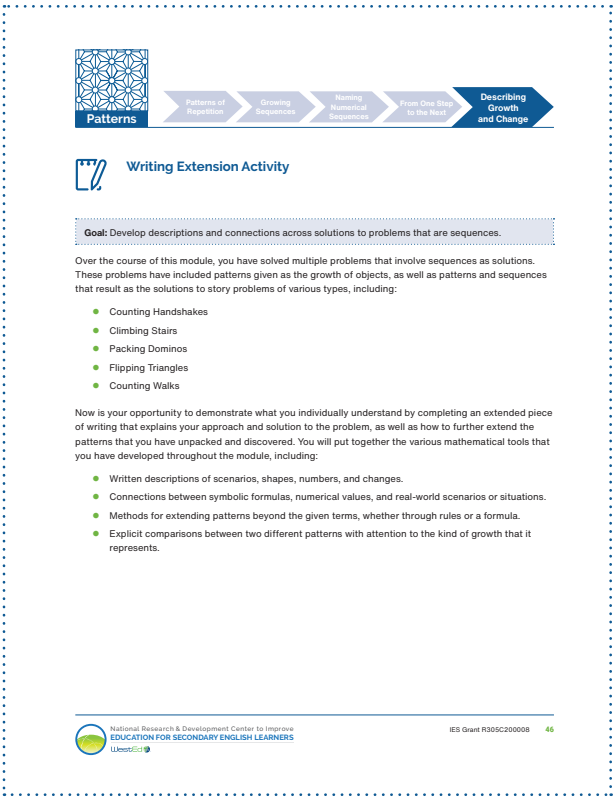


Close and Connect

5. Ask students to share what they found as a generalizable pattern for any width. Students can post their ideas for other groups to see. Ask students to circle the room and review the work of other groups.
6. Then, discuss: *What steps did you take to identify a generalizable pattern? Which steps were key to your problem-solving approach?*
7. Close with a discussion of any differences in patterns or approaches that students took.

Writing Extension Activity

Students will develop descriptions and connections across solutions.


Patterns
Patterns of Repetition | Growing Sequences | Naming Numerical Sequences | From One Step to the Next | Describing Growth and Change

Writing Extension Activity

Goal: Develop descriptions and connections across solutions to problems that are sequences.

Over the course of this module, you have solved multiple problems that involve sequences as solutions. These problems have included patterns given as the growth of objects, as well as patterns and sequences that result as the solutions to story problems of various types, including:

- Counting Handshakes
- Climbing Stairs
- Packing Dominos
- Flipping Triangles
- Counting Walks

Now is your opportunity to demonstrate what you individually understand by completing an extended piece of writing that explains your approach and solution to the problem, as well as how to further extend the patterns that you have unpacked and discovered. You will put together the various mathematical tools that you have developed throughout the module, including:

- Written descriptions of scenarios, shapes, numbers, and changes.
- Connections between symbolic formulas, numerical values, and real-world scenarios or situations.
- Methods for extending patterns beyond the given terms, whether through rules or a formula.
- Explicit comparisons between two different patterns with attention to the kind of growth that it represents.

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Purpose and Rationale

The purpose of this task is for students to synthesize their understandings of patterns across written descriptions, connections, and comparisons.

Prepare

1. Have students turn to the *Writing Extension Activity Rubric* in their binders.
2. Go over the sections covering the *Content and Design* expectations.

Enact and Observe

3. Have students develop their writing responses around the following:
    - Written descriptions of scenarios, shapes, numbers, and changes.
    - Connections between symbolic formulas, numerical values, and real-world scenarios or situations.
    - Methods for extending patterns beyond the given terms, whether through rules or formulas.
    - Explicit comparisons between two different patterns with attention to the kind of growth that it represents.

Close and Connect

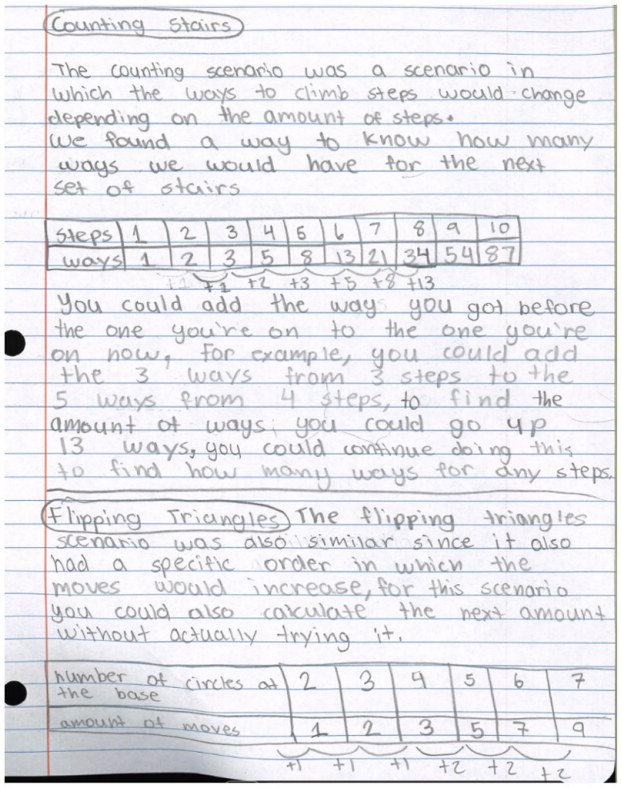
4. Have students share ideas they are developing to help each other to help refine their draft.

Writing Extension Activity Rubric

Category	Indicators of High Quality Work	Strengths	Areas to Improve or Revise
Content	• Written descriptions provide detail that enables connections about numbers, shapes, and changes. • Analysis of changes in the sequence of answers are connected to the meanings in real-world scenarios. • Information or an approach is developed to extend the pattern. • Explanations explicitly compare and contrast different patterns in terms of growth and meaning.		
Design	• Use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

Sample Student Writing Extension Activities

Elena identified differences in the sequences and in her strategies.



Counting Stairs

The counting scenario was a scenario in which the ways to climb steps would change depending on the amount of steps. We found a way to know how many ways we would have for the next set of stairs

steps	1	2	3	4	5	6	7	8	9	10
ways	1	2	3	5	8	13	21	34	54	87

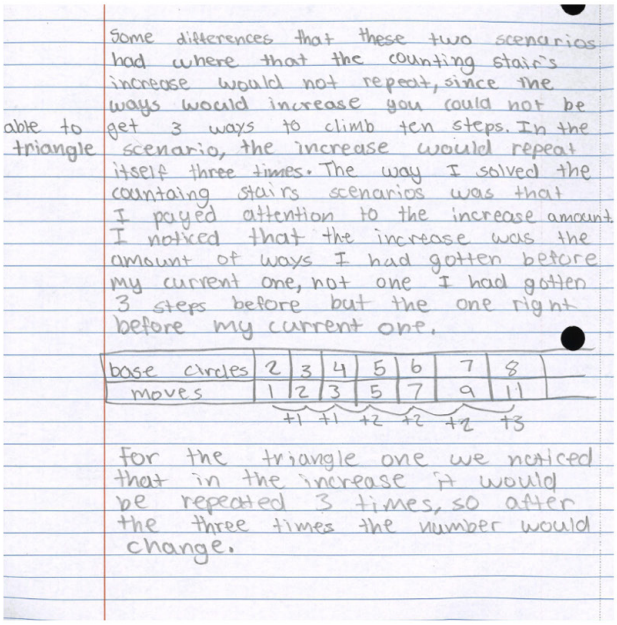
+1 +2 +3 +5 +8 +13

You could add the way you got before the one you're on to the one you're on now, for example, you could add the 3 ways from 3 steps to the 5 ways from 4 steps, to find the amount of ways; you could go up 13 ways, you could continue doing this to find how many ways for any steps.

Flipping Triangles The flipping triangles scenario was also similar since it also had a specific order in which the moves would increase, for this scenario you could also calculate the next amount without actually trying it.

number of circles at the base	2	3	4	5	6	7
amount of moves	1	2	3	5	7	9

+1 +1 +1 +2 +2 +2



able to triangle

Some differences that these two scenarios had where that the counting stairs increase would not repeat, since the ways would increase you could not be get 3 ways to climb ten steps. In the scenario, the increase would repeat itself three times. The way I solved the countaing stairs scenarios was that I payed attention to the increase amount I noticed that the increase was the amount of ways I had gotten before my current one, not one I had gotten 3 steps before but the one right before my current one.

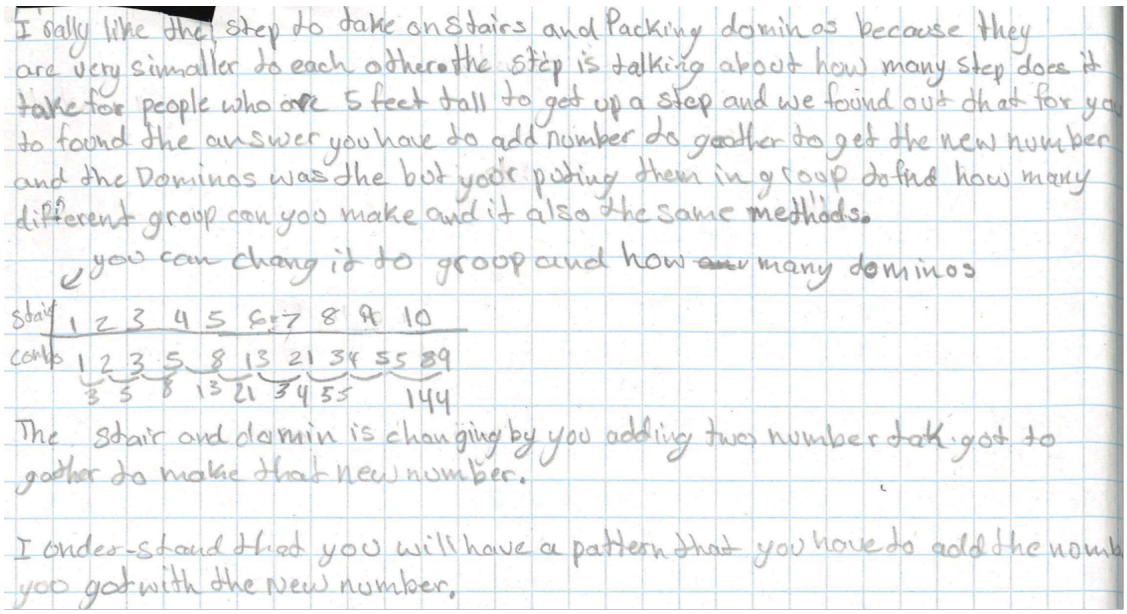
base circles	2	3	4	5	6	7	8	
moves	1	2	3	5	7	9	11	

+1 +1 +2 +2 +2 +3

for the triangle one we noticed that in the increase it would be repeated 3 times, so after the three times the number would change.

Sample Student Writing Extension Activities

Opema noted similarities in the sequences for the *Dominos* and *Climbing Stairs* problems.



I really like the step to take on Stairs and Packing dominos because they are very simaller to each other. the step is talking about how many step does it take for people who are 5 feet tall to get up a step and we found out that for you to found the answer you have to add number to gether to get the new number and the Dominos was the but your puting them in group to find how many different group can you make and it also the same methods.

you can chang it to groop and how many dominos

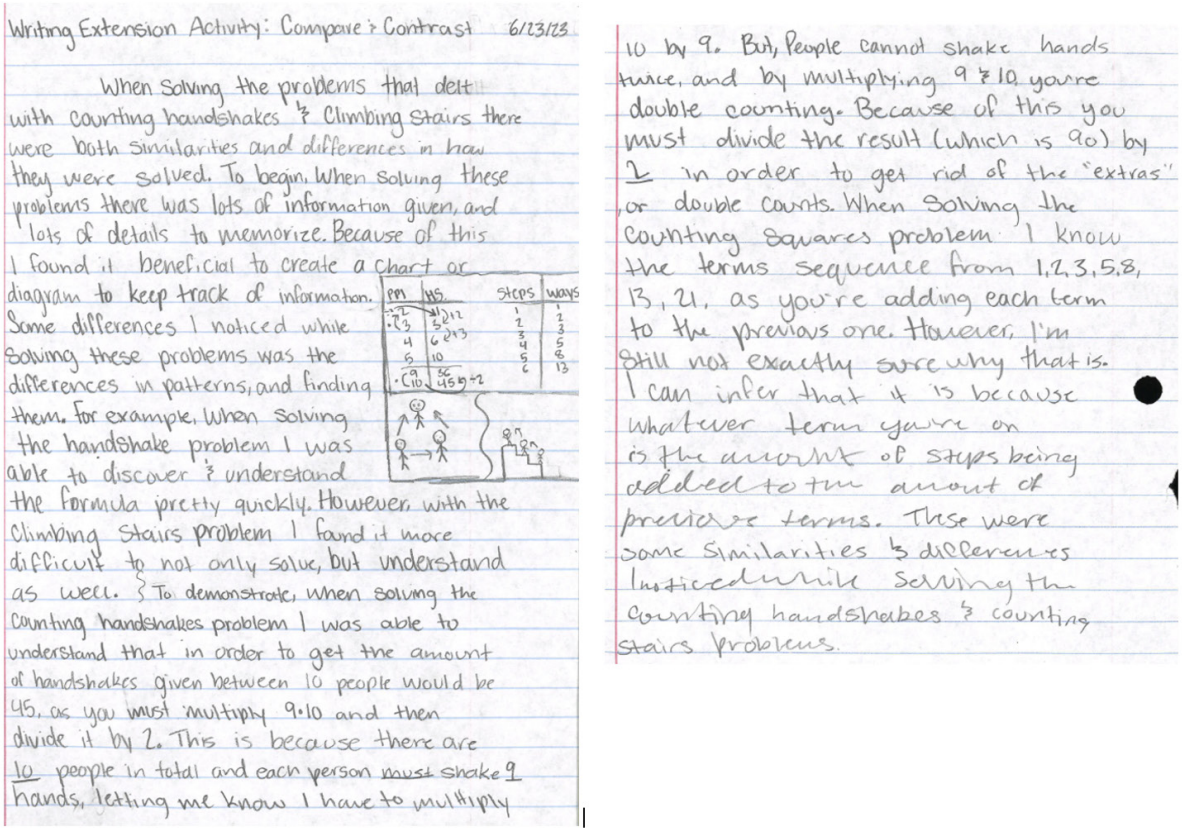
stair	1	2	3	4	5	6	7	8	9	10
combs	1	2	3	5	8	13	21	34	55	89
	3	5	8	13	21	34	55		144	

The stair and domin is changing by you adding two number tak got to gether to make that new number.

I onder-staud that you will have a pattern that you have to add the numb you got with the new number.

Sample Student Writing Extension Activities

Lola described her own successes and challenges when identifying the patterns in the *Counting Handshakes* and *Climbing Stairs* problems.



Writing Extension Activity: Compare & Contrast 6/23/23

When solving the problems that delt with counting handshakes & Climbing Stairs there were both similarities and differences in how they were solved. To begin, When solving these problems there was lots of information given, and lots of details to memorize. Because of this I found it beneficial to create a chart or diagram to keep track of information. Some differences I noticed while solving these problems was the differences in patterns, and finding them. For example, When solving the handshake problem I was able to discover & understand the formula pretty quickly. However, with the Climbing Stairs problem I found it more difficult to not only solve, but understand as well. To demonstrate, when solving the Counting handshakes problem I was able to understand that in order to get the amount of handshakes given between 10 people would be 45, as you must multiply 9·10 and then divide it by 2. This is because there are 10 people in total and each person must shake 9 hands, letting me know I have to multiply 10 by 9. But, People cannot shake hands twice, and by multiplying 9 & 10 you're double counting. Because of this you must divide the result (which is 90) by 2 in order to get rid of the "extras" or double counts. When solving the Counting Squares problem I know the terms sequence from 1, 2, 3, 5, 8, 13, 21, as you're adding each term to the previous one. However, I'm still not exactly sure why that is. I can infer that it is because whatever term you're on is the amount of steps being added to the amount of previous terms. These were some similarities & differences I noticed while solving the Counting handshakes & counting stairs problems.

Reflect After Day 5

Please reflect on student learning by responding to the following questions:

1. What strategies did students use for the *Flipping Triangles* task? Were any of their strategies unexpected?

2. For which students did the *Bookmark for Planning for a Solution* facilitate their planning for their solution? How do you know?

3. What, if any, changes in student mathematical talk did you notice across the module?

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