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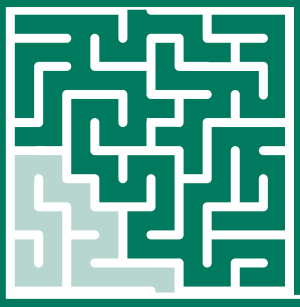


TEACHER MANUAL

Equivalence & Transformation:
Can you get from here to there?

Module Overview

This module invites students to explore and connect multiple instances of equivalence as it is related to a set of transformations. In mathematics, the key concept of equivalence is how a set of objects are the same "up to" a set of transformations. For example, in geometry, two shapes are *congruent* if they are connected by *rigid motions*. Meanwhile, two rates of change are *equivalent* if they are the same up the *scaling*. In this module, students explore examples that cross the domains of algebra, statistics, and geometry and develop deeper understanding of a selected case. For English Learners, this module provides explicit models for how to describe sequences of transformations, look for underlying structure, and develop arguments for whether objects are equivalent.

Conceptual Understandings	Mathematical Practices	Language Practices
• Equivalence is related to a set of permissible transformations. • An equivalence relation partitions a set into classes. • New mathematical objects can be created through equivalence.	• Reason abstractly and quantitatively (MP2). • Model with mathematics (MP4). • Attend to precision (MP6).	• Describe transformations between objects. • Verify key properties of equivalence. • Trace what changes and what remains the same across transformations.

Overview of Content and Objectives

The arrow below displays how students will begin by exploring equivalence through familiar examples: scaling to create equivalent rates. The activities in this module deepen students' conceptual understanding of equivalence across the domains of algebra, geometry, and statistics. The activities also facilitate student learning of key analytical practices, including the Standards for Mathematical Practice listed in the above table. Sequences of activities are offered to students as they extend their language use around explanation, a key practice in mathematics.

	Mazes and Mean	Slides and Codes	Perspective Geometry	Extending Equivalence
Objectives	• Develop a working definition of equivalence and transformation • Explore equivalence classes of objects within a particular domain • Identify special representatives within each equivalence class • Explain how objects can be created through equivalence and transformations			
Tasks at a Glance	• Can you get there from here? (And Back!) • Create your own maze • Compare and Contrast Matrix • Novel Ideas Only • Think Pair Share • Human Number Line • Step it Up • Sort and Label • Compare and Contrast Matrix • Daily Writing Prompt	• Explore Slide Puzzles • Compare and Contrast Matrix • Alphabetic Codes • Reading with Clarifying Bookmark III • Partner Encryption • Sort and Label • Create-Exchange-Assess • Compare and Contrast Matrix • Daily Writing Prompt	• Perspective Geometry w/ Clarifying Bookmark • Step It Up • Compare and Contrast Matrix • Daily Writing Prompt	• Writing Extension Activity • Final Reflection • Math and Me Assessment

Key Considerations

How to Group Students

In many of these activities, students will work with a partner or in a small group. There are many ways of how to group students into teams. Students can be grouped by interests, mathematics ability, compatibility, or convenience. There are potential advantages and drawbacks with different ways to group students. For example, convenience grouping, when a teacher asks students who sit near each other already to work together, is easy to organize, but student groups may not have students with different interests or abilities. Grouping students with a variety of interests or abilities can bring together student strengths, but these groups may be harder to manage. Consider different ways to group students before each activity and intentionally choose a grouping strategy.

Attention to Language Development

One of the key uses of language developed through this module is the verification of properties of equivalence as well as the explanation of cases of equivalence using both representatives and relationships. The thread of explaining equivalence unfolds over the following tasks in this module:

- ***Novel Ideas Only:*** Students generate ideas about "balance" that serve as the basis for later refinements, including examples that are potentially more on the side of equal or identical rather than the instances that will require equivalence in a transformative sense.
- ***Step it Up:*** As pairs of students work together, they engage in two connected processes. First, they respond to each other's transformations to ensure balance is maintained. Second, as partners they chain together transformations to co-create sequences of equivalent objects that may take unpredictable pathways and end in interesting final products. These experiences with the assistance of a partner will assist students as they construct their own chains of transformations independently and then the explanations of those operations.
- ***Sort and Label:*** Students apply their understanding of equivalent means by grouping different data sets together and co-creating descriptions that can characterize the groups of cards by similar traits. In creating groups based on equivalence they are also able to discuss what nonequivalent relationships are among the data sets.
- ***Think Pair Share:*** This activity offers students the opportunity to see different situations that focus on two sides that are either balanced or not. Students will be able to see how the center of gravity is the point of balance, that the distance from the balance matters, and that the size or weight of each rock also matters.

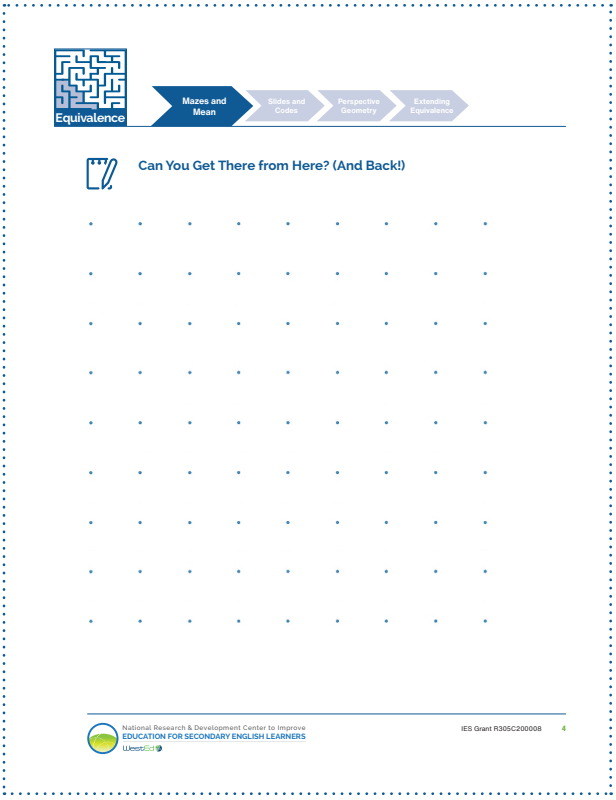
- ***Compare and Contrast Matrix:*** Students will deepen their understanding of equivalence by answering cross cutting questions about equivalence and transformation among the different math activities they will experience.

As students engage in these activities, they are challenged and supported in their language development along three dimensions:

- They grow from more interactive, dialogic interactions to negotiated descriptions that develop into more monologic, edited, and rehearsed descriptions.
- Students move from working together with peers to figure something out together toward becoming more expert and authoritative in their uses of language.
- The language that students use to refer to the objects that they are discussing moves from more everyday uses toward more technical uses, appropriate for developing deeper mathematical ideas and practices.

Can You Get There from Here? (And Back!)

Puzzles such as mazes offer an opportunity for students to consider different conditions of equivalence. In this activity, students explore mazes, write through what is equivalent, and reflect on representations of equivalence.



Purpose and Rationale

The purpose of this task is for students to identify and apply properties that determine equivalence. When students explore mazes, they can see connected components and evidence of equivalence in a puzzle that may be a familiar object or one that students did not initially think of as having conditions of equivalence.

Prepare

1. Watch the video, *Can You Get There from Here? (And Back!),* and try the task of creating mazes with different connected components.
2. Anticipate what students may try or have difficulty seeing when they make their own mazes and exchange them with each other.

Enact (what to do) and Observe (what to look/listen for)

3. Distribute the *Can You Get There from Here? (And Back!)* handout. Ask students to watch and discuss the video with a partner.
4. Students will then use the dot paper to create a maze individually and label some points. Students will exchange papers with their partner and try to separate the maze into different connected components. Encourage them to use key terms such as symmetry and equivalent in their writing.
5. As students are working, identify students to share their reasoning about how they created connected components. As the purpose is to highlight the diversity of ideas, choose a set of three or four students who use different language to describe equivalence.

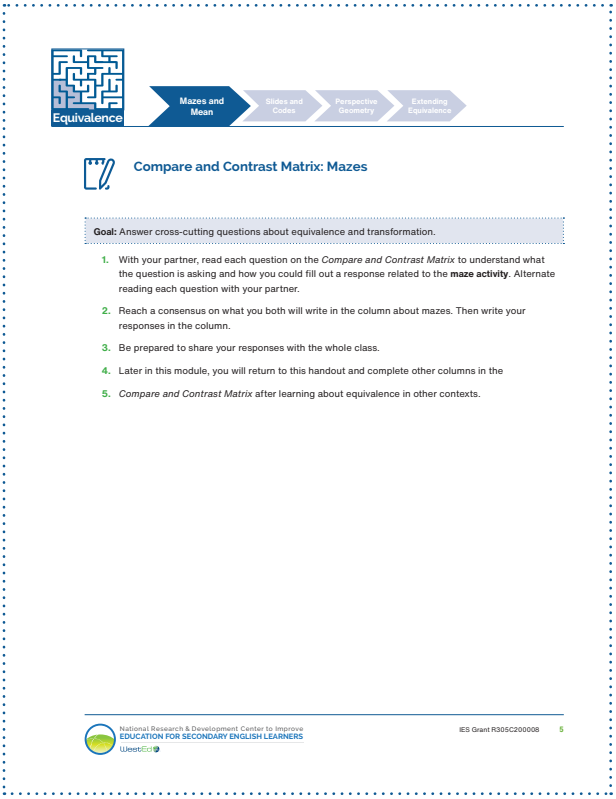
Close and Connect

6. Close the activity by asking the students that you chose to share their mazes and the connected components. Highlight how equivalence is present in different mazes.
7. Then, tell students that the next activity will involve sharing your ideas on how quantities in real-world settings are related.

Compare and Contrast Matrix: Mazes

Purpose and Rationale

The purpose of this task is for students to identify the concepts of equivalence and transformations found within the Mazes task. When students compare different problems and identify what transformations are allowed and not allowed based on their individual properties, they will reinforce their understanding of equivalence and therefore be able to communicate what the concept of equivalence is.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Mazes

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, read each question on the *Compare and Contrast Matrix* to understand what the question is asking and how you could fill out a response related to the **maze activity**. Alternate reading each question with your partner.
2. Reach a consensus on what you both will write in the column about mazes. Then write your responses in the column.
3. Be prepared to share your responses with the whole class.
4. Later in this module, you will return to this handout and complete other columns in the
5. *Compare and Contrast Matrix* after learning about equivalence in other contexts.

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Prepare

1. Organize students in pairs. Distribute one copy of the *Compare and Contrast Matrix* to each student.
2. Have each student read aloud each question on the first column to understand the question and how the matrix should be completed. Partners should alternate reading the questions.

Enact and Observe

3. Partners should write their responses individually about mazes. Then, partners will share what they wrote and resolve any conflict.
4. Remind students that they should be prepared to share what their partners have discussed.

Close and Connect

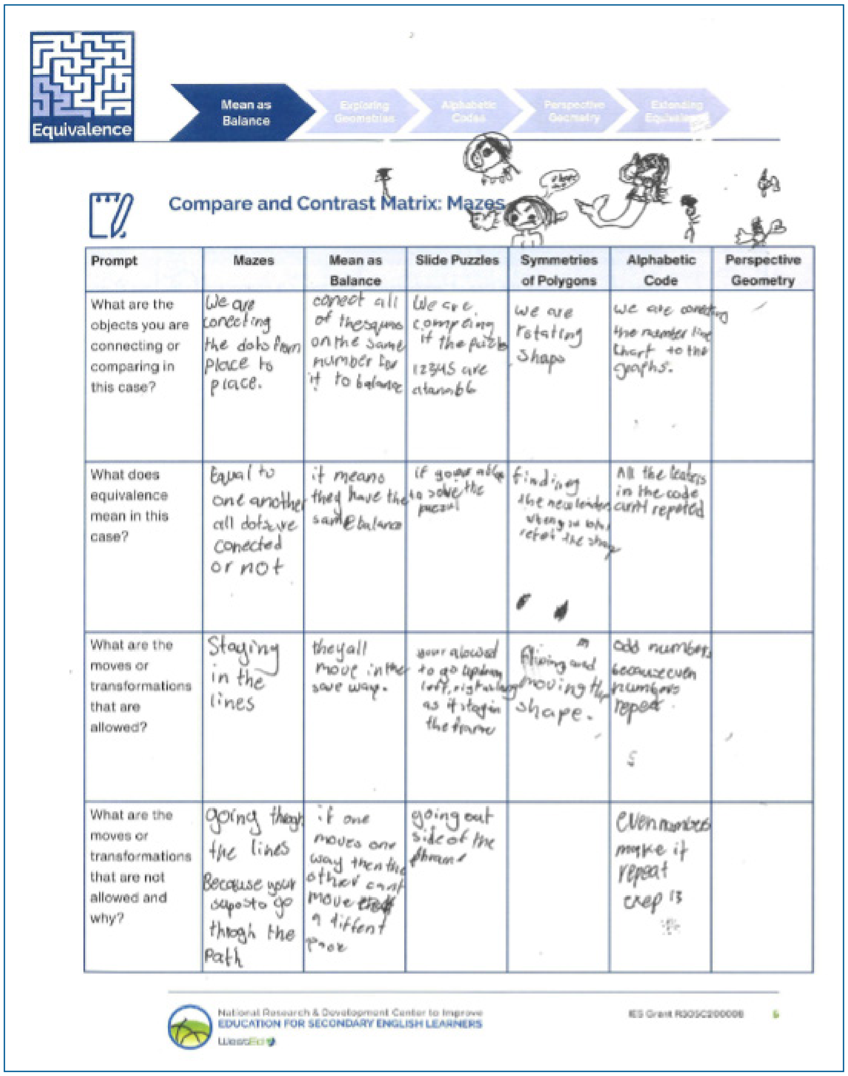
5. Once they have completed the matrix, hold a whole class discussion where you elicit participation from the class to share what it means for two locations to be equivalent in the mazes and how they know.

Compare and Contrast Matrix: Mazes

Prompt	Mazes	Mean as Balance	Slide Puzzles	Alphabetic Code	Perspective Geometry
What different types of objects did you compare and connect?					
What moves can you make to maintain equivalence?					

Prompt	Mazes	Mean as Balance	Slide Puzzles	Alphabetic Code	Perspective Geometry
What moves can you not make, and why?					
What other ways could check whether two objects are equivalent?					

Student Work Sample



Equivalence

Mean as Balance | Exploring Geometries | Alphabetic Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Mazes

Prompt	Mazes	Mean as Balance	Slide Puzzles	Symmetries of Polygons	Alphabetic Code	Perspective Geometry
What are the objects you are connecting or comparing in this case?	We are conecting the dots from place to place.	conect all of the squares on the same number for it to balance	We are comparing if the puzzle 12345 are [illegible]	we are rotating shaps	we are conecting the number line chart to the graphs.	
What does equivalence mean in this case?	Equal to one another all dots are conected or not	it means they have the same balance	if [illegible] to solve the puzzel	finding the [illegible] [illegible] the shape	All the leaters in the code can't repeted	
What are the moves or transformations that are allowed?	Staying in the lines	they all move in the save way.	your alowed to go up down left, right [illegible] as it stay in the frame	flipping and moving the shape.	odd numbers because even numbers repeat	
What are the moves or transformations that are not allowed and why?	going through the lines Because your supposto go though the path	if one moves one way then the other can't move [illegible] a diffent [illegible]	going out side of the frame		even numbers make it repeat [illegible]	

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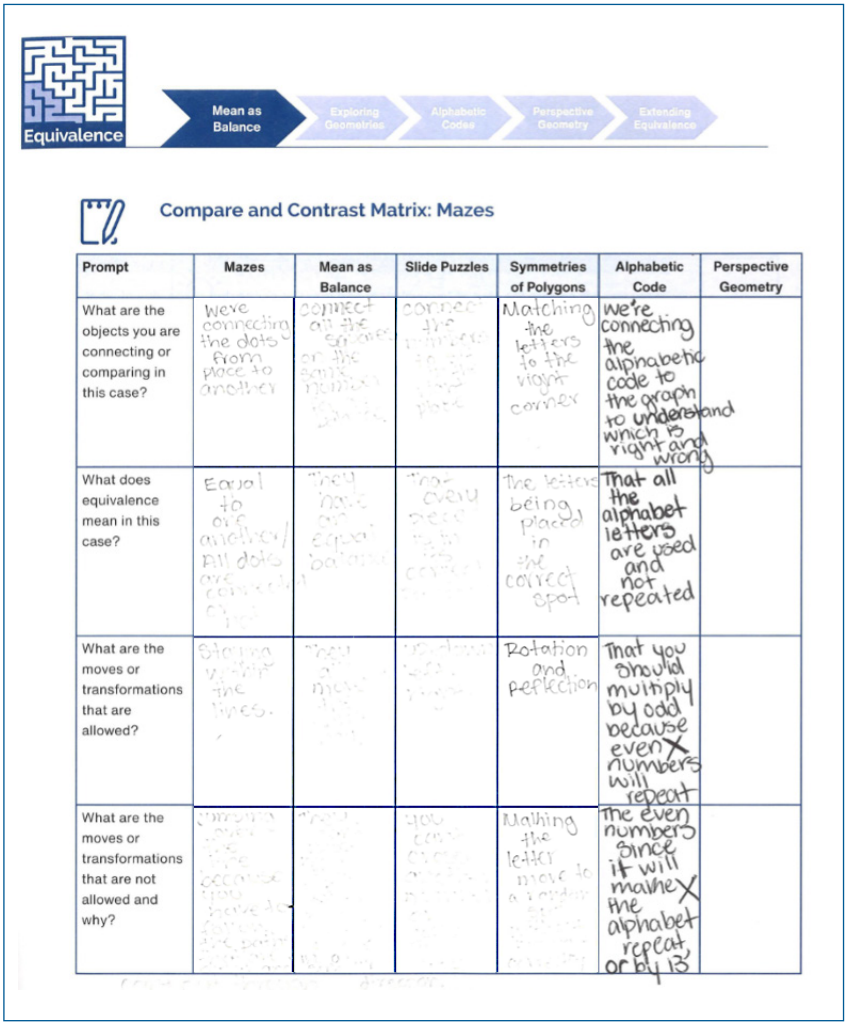
Student Work Sample



Equivalence

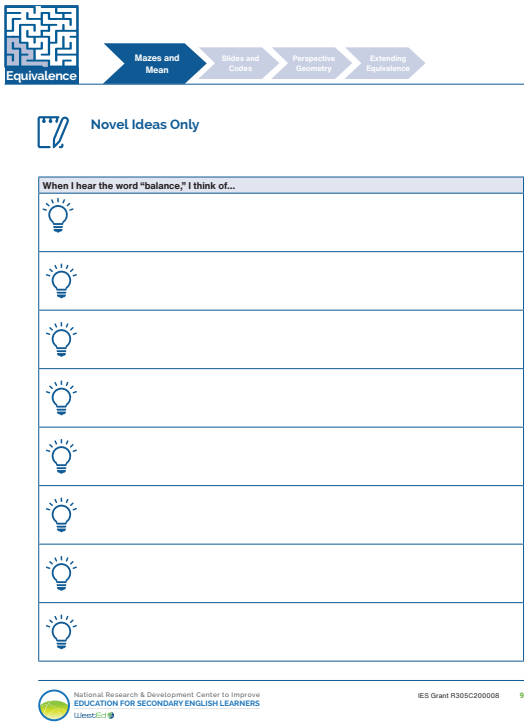
Mean as Balance | Exploring Geometries | Alphabetic Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Mazes

Prompt	Mazes	Mean as Balance	Slide Puzzles	Symmetries of Polygons	Alphabetic Code	Perspective Geometry
What are the objects you are connecting or comparing in this case?	We've connecting the dots from place to another	connect all the	connect the	Matching the letters to the right corner	We're connecting the alphabetic code to the graph to understand which is right and wrong	
What does equivalence mean in this case?	Equal to or another/ All dots are connected			The letters being placed in the correct spot	That all the alphabet letters are used and not repeated	
What are the moves or transformations that are allowed?	staying the lines.			Rotation and reflection	That you should multiply by odd because even numbers will repeat	
What are the moves or transformations that are not allowed and why?				Mathing the letter	The even numbers since it will make the alphabet repeat, or by 13	

Novel Ideas Only

The *Novel Ideas Only* task elicits what students already know about the word "**balance**" and emphasizes the valuable prior knowledge that they bring to this unit.



Purpose and Rationale

The purpose of this activity is to surface students' prior knowledge and experiences regarding equivalence in relation to their prior understanding of what may or may not be balanced. This activity provides an opportunity for students and the teacher to hear students' current ideas about "**balance**" and equivalence. Use this activity to recognize and connect to students' ideas before inviting them to expand their skills and knowledge in subsequent activities. The small group structure provides an opportunity for students to share their ideas with their peers; then, in the full group, students hone their listening skills as other groups present. As you enact this activity, be ready to listen carefully for student ideas that you may want to highlight and encourage during later activities. Connecting later learnings to students' current ideas can help students make connections and build a robust understanding of patterns.

Remember that the goal of the activity is to collect students' ideas and encourage their thinking around balance as it relates to equivalence; the goal is not to write out an exhaustive list of all ideas about same or equivalence. Refrain from asking students to write all ideas they hear.

Prepare

1. Before you begin, note at least three ideas that you anticipate your students use to complete the prompt, "When I hear the word "**balance**," I think of ..." Note your ideas here:
   -
   -
   -
2. Review both parts of this activity and determine how you will group students.

Enact and Observe

3. Distribute the *Novel Ideas Only* handout to each student. Read the directions for small group work brainstorming.
4. Give students 3 minutes for small group work. In each group, one student will share an idea in response to the prompt, another student will repeat it, and all students will write it down. Students continue sharing, repeating, and writing ideas for 3 minutes.
5. After students work in groups for 3 minutes, lead a full sharing where students Equivalence & Transformation: Can you get from here to there? report out their "novel ideas:" Ask one student from one group to stand and read all the ideas that their group collected. Other students should listen carefully and fill in the lightbulb if one of their ideas is read aloud. Then, the next group should read aloud only the ideas from their group that have not yet been shared. Continue this process until all groups have shared their novel ideas.
6. After students have completed reporting, encourage them to keep listening, and to add to their list if they hear a new idea that they would like to include

Close and Connect

7. Close the activity by thanking students for reporting out their novel ideas and then sharing a theme that you heard across student responses; for example, how balance relates to the sameness of one quantity to another, hence making something balanced. Highlight language, such as equivalence or equality, that you have heard students use that will be useful in later activities
8. Then, tell students that in the next activity they will explore images around equivalence and how they should begin to think about how conditions that make objects equivalent connecting with the ideas they have shared so far.

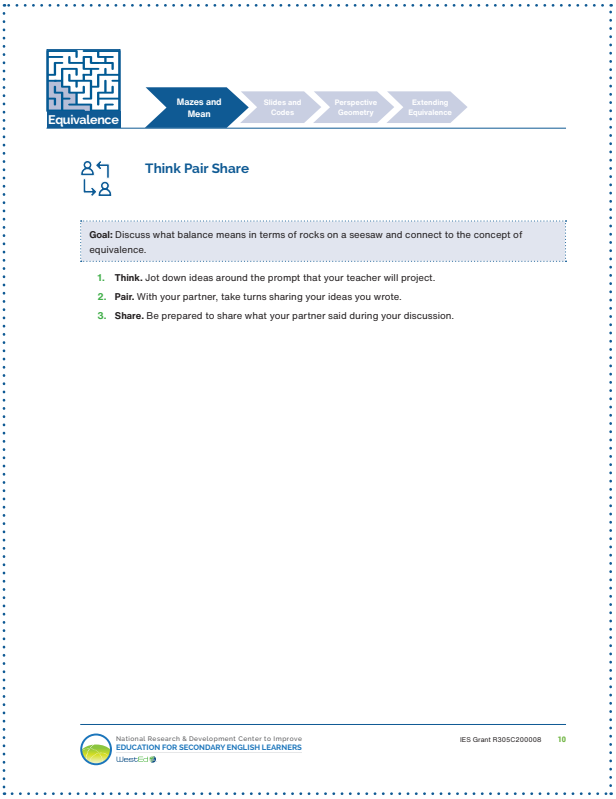
Novel Ideas Only

When I hear the word "balance," I think of ...

Think Pair Share

Purpose and Rationale

The purpose of this task is to surface students' prior knowledge understanding of balance as it connects to the concept of equivalence. This activity offers students the opportunity to see different situations that focus on two sides that are either balanced or not. Students will be able to see how the center of gravity is the point of balance, that the distance from the balance matters, and that the size or weight of each rock also matters.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Think Pair Share

Goal: Discuss what balance means in terms of rocks on a seesaw and connect to the concept of equivalence.

1. **Think.** Jot down ideas around the prompt that your teacher will project.
2. **Pair.** With your partner, take turns sharing your ideas you wrote.
3. **Share.** Be prepared to share what your partner said during your discussion.

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Prepare

1. Prepare the PowerPoint slides with images of the rocks on the seesaw.
2. Tell students that they will have two minutes to jot down ideas individually in response to the following prompt:
    - Select one of the images and decide whether you think it would actually be balanced or not. Give reasons why.

Enact and Observe

3. Arrange students in pairs.
4. Instruct students that they will take turns sharing with each other what they wrote about. Tell students that they will then share with the whole class what his or her partner said.

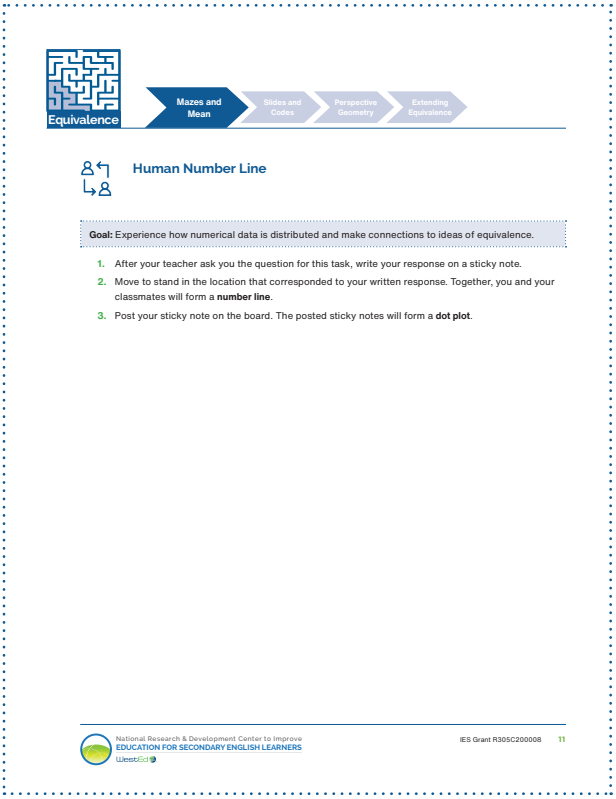
Close and Connect

5. Call on a few students to share what their partners said with the whole class. Lead a whole class discussion about what it means to balance and how equivalence plays a role in the images. Possible prompts are: When the image was balanced, what about it was equivalent? How did you know that it was equivalent? Use student ideas to help define equivalence related to balance.

Think Pair Share: Balancing Images

Human Number Line

The *Human Number Line* task allows students to take data gathered from their responses and plot it on a line plot to better understand how data from the real world can be represented as data on a number line.



Purpose and Rationale

The purpose of this task is to understand how numerical data is distributed across a continuum. In this activity, students take numerical data from the responses to questions they were asked and transform this data onto a number line to begin to understand the properties of equivalence and transformation.

Prepare

1. Prepare a long strip of paper by taping together poster paper or printer paper so that it stretches the length of the classroom or at least several feet.

Enact and Observe

2. Draw a number line on this paper, corresponding to the questions you will ask. Suggestion Questions you can ask:
    - How far do you live from school?
    - What is your height in inches?
    - How many people live in your home?
3. Ask students: Which have both integer answers as well as answers that will be rational or real numbers. They will write their responses on a sticky note.

While students are working, prompt them to focus on how the data is distributed on the Human Number line and what properties of equivalence they may have as they post their stickies on the board.

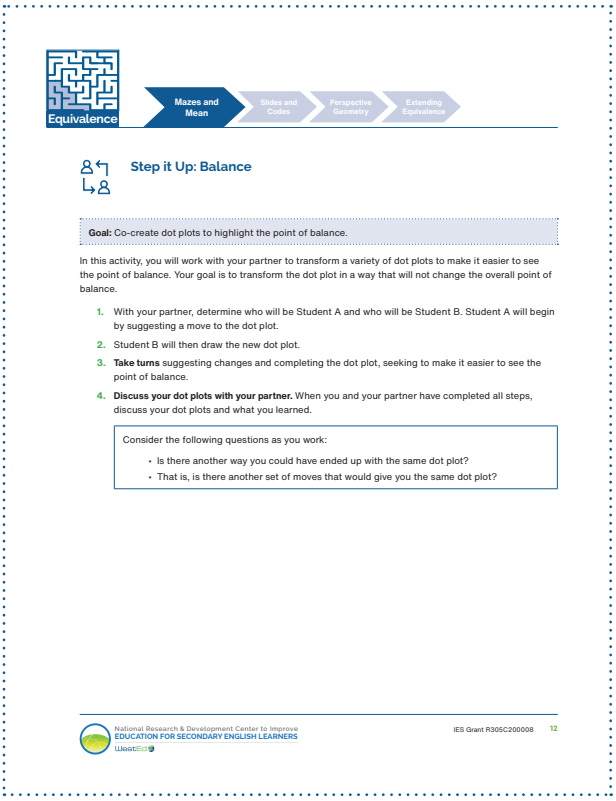
4. Have students physically move to the locations
5. They then post their sticky notes on the board to replicate this idea from the Human Number line to the line plot on the board.

Close and Connect

6. Close the activity by having students share their response to how equivalence can be seen in this *Human Number Line* task. Note similarities and differences in how students are talking about equivalence, that is, if students are emphasizing transformations or relations when talking about equivalence in this task and even as compared to talk about equivalence in the earlier images.

Step it Up: Balance

Students co-create line plots that have the data points distributed more centrally.



Purpose and Rationale

In the *Step it Up* task, students explore how changes to the distribution of data points can preserve the mean to calculate it more easily. They will deepen their understanding of equivalence as they will be able to compare different line plots with the same mean but with different distributions of data. By creating line plots themselves, students can explore the process as well as the final product and can investigate how to arrive at line plots that have the same mean in more than one way.

Prepare

1. Determine how you will group students for this activity.
2. Review the activity and take note of how students could maintain the mean through a different sequence of steps.

Enact and Observe

3. Organize students into pairs and distribute the *Step It Up Handout Cards* with the initial line plots and the *Step it Up Recording Sheet*. These should be cut out ahead of time. Have students decide who will be Partner A and who will be Partner B. Have students recreate the chosen line plot on the recording sheet.
4. With one student, model the first two turns of a move, writing the description of change, and then drawing the new line plot based on that change. Ask students to follow along but not copy down this sample.

5. As students begin to work with a partner on the line plots in the *Step It Up Cards* encourage them to share their thoughts around **equivalence** as it relates to the changes they are suggesting from one move to the next.

> Consider the following questions as you work:
>
> - Is there another way you could have ended up with the same dot plot?
> - That is, is there another set of moves that would give you the same dot plot?

Close and Connect

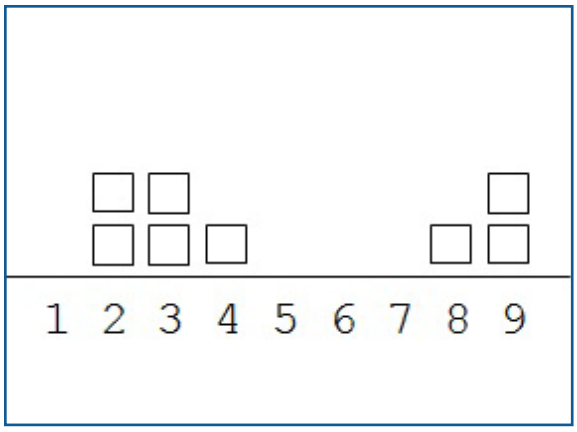
6. Close the activity by having some students show their work, including the initial and the final result. Lead a discussion in which you demonstrate the standard algorithm by observing how for each of the equivalent dot plots the total and the number of data points does not change. Elicit from a student the idea that one way to compute the average is to take the total and divide by the number of data points, which is assuming that all points have the same value.

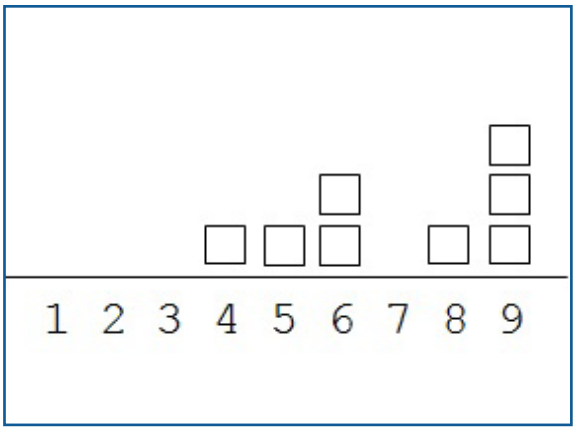
Mean as Balance Step it Up Recording Sheet

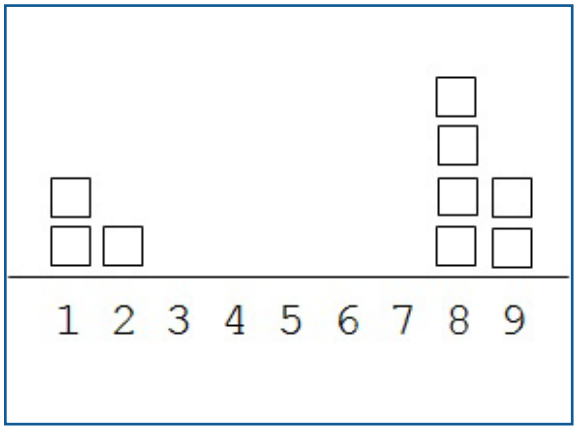
Partner	Changes	Responses	Dot Plot
			1 2 3 4 5 6 7 8 9
A			1 2 3 4 5 6 7 8 9
B			1 2 3 4 5 6 7 8 9
A			1 2 3 4 5 6 7 8 9
B			1 2 3 4 5 6 7 8 9

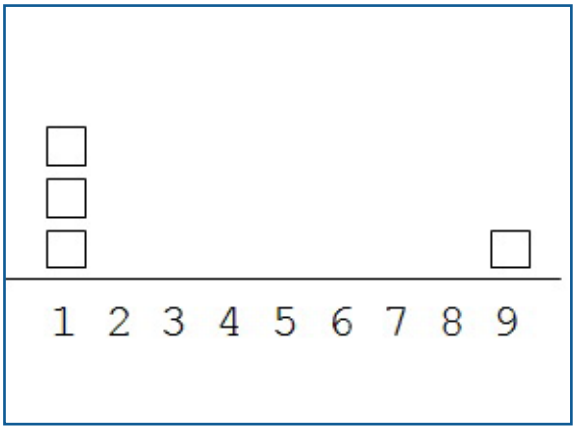
Partner	Changes	Responses	Dot Plot 1 2 3 4 5 6 7 8 9
A			
B			

Step It Up Cards



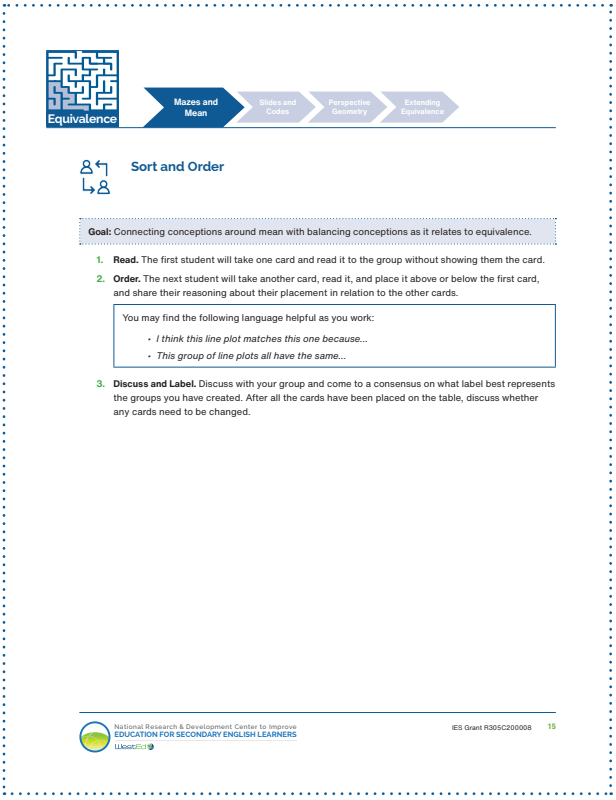






Sort and Label

There are different types of lines plots based on the distribution of data, that have a variety of mathematical relationships present in them. In this activity, students describe the attributes that can define mean and order multiple data sets into groups by similar traits.



Purpose and Rationale

The purpose of this task is to have students focus on the number of data values and their distribution in connecting conceptions around mean with balancing conceptions as it relates to equivalence. In this activity, students focus on how the data is distributed across the line plots and use this information to then compare them with one another.

Prepare

1. Identify different ways that students may sort the line plots based on the different distributions of data. Note at least two different ways that you could compare line plots that are distributed differently.

Enact and Observe

2. Organize students into groups of four. Distribute the *Sort and Label Cards.*
3. Read the directions for the task aloud. Tell students that they will sort the cards one at a time, discussing the cards in terms of the total number of data points and the values, and then tentatively suggesting how to sort the cards. On each turn, a student will take a card and describe it to the group without showing them the card. In subsequent turns, students will describe their cards and their method for comparing it to the other line plot, then suggest whether the new card belongs in the same group to the other cards.

4. Make note of how students sort and label the groups of cards. Prompt student conversations with the formulaic expressions.
5. If students ask help, encourage them to either estimate a point of balance or utilize what they learned in the *Step It Up* activity to find a point of balance.
6. After all the cards are placed on the table, students can discuss whether the labels need to be changed or refined to better describe the groups of cards.

Close and Connect

7. Close the activity by sharing two samples of how cards were sorted then labeled using the point of balance.
8. Project the following prompt on the white board and hold a whole class discussion connecting on concepts around equivalence and transformations that can lead to equivalence:
    - How can we see properties of equivalence and/or transformations that lead to equivalence in this *Sort and Label* task?

While students are working, prompt them to focus on how the data is distributed and what properties of equivalence they may have. For example, one line plot may have data distributed evenly across the number line while another may have data in clusters around a point. Ask students to describe how they are finding equivalence and the importance of thinking through their point of balance and other concepts previously discussed around equivalence.

Share these formulaic expressions for students to use during small group work:

- *I think this line plot matches this one because ...*
- *This group of line plots all have the same ...*

Sort and Label

Goal: Connecting conceptions around mean with balancing conceptions as it relates to equivalence.

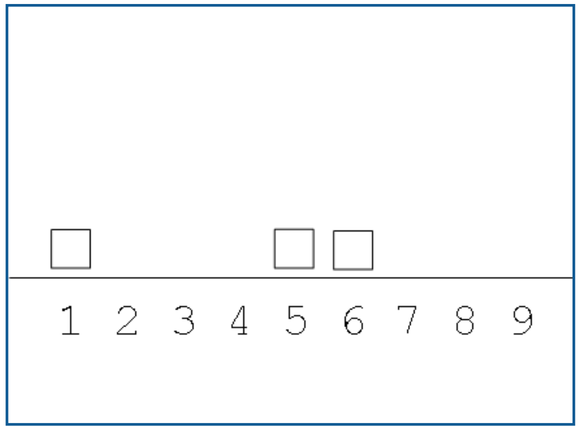
1. **Read.** The first student will take one card and read it to the group without showing them the card.
2. **Order.** The next student will take another card, read it, and place it above or below the first card, and share their reasoning about their placement in relation to the other cards.

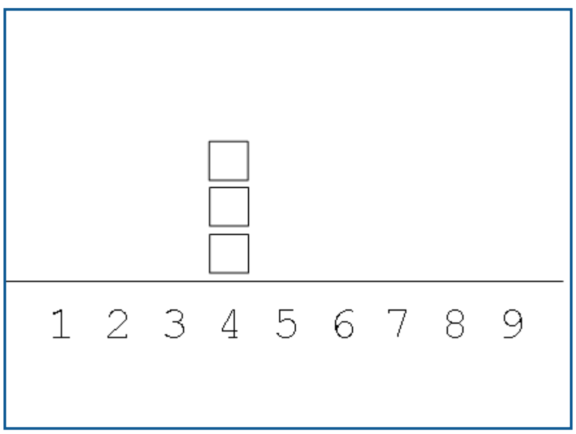
   You may find the following language helpful as you work:

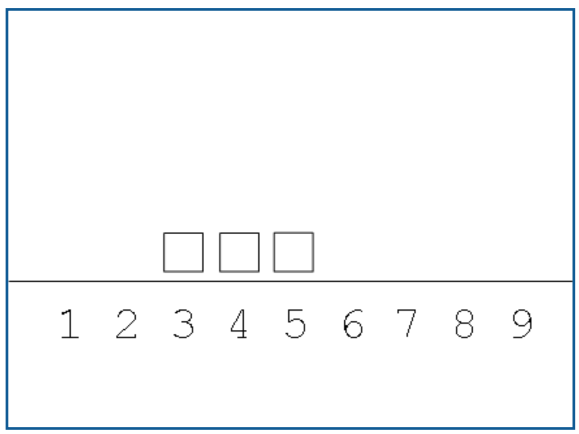
   - *I think this line plot matches this one because ...*
   - *This group of line plots all have the same ...*

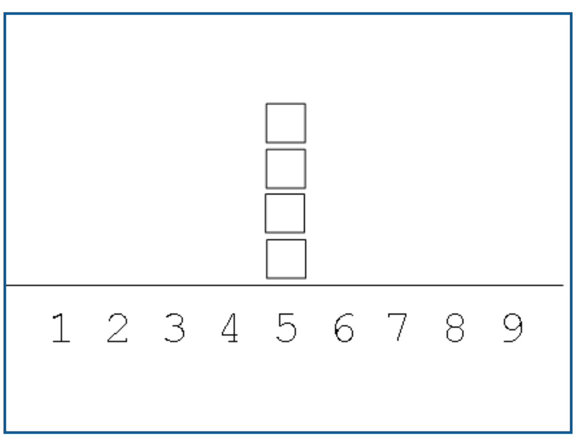
3. **Discuss and Label.** Discuss with your group and come to a consensus on what label best represents the groups you have created. After all the cards have been placed on the table, discuss whether any cards need to be changed.

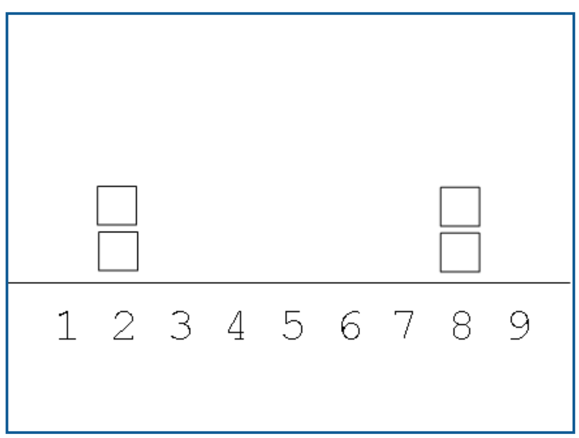
Sort and Label Cards

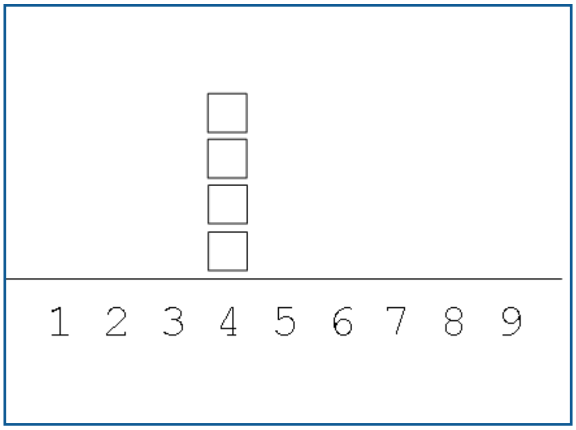








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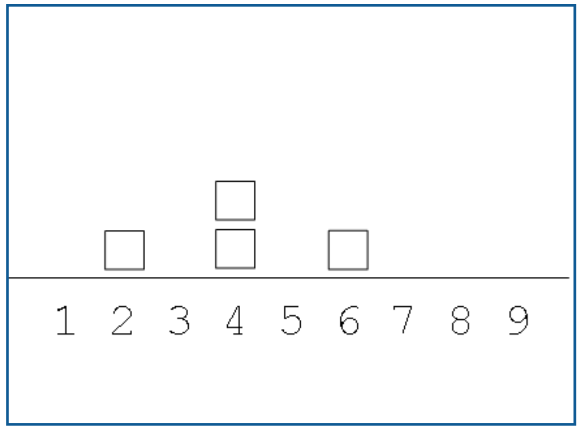
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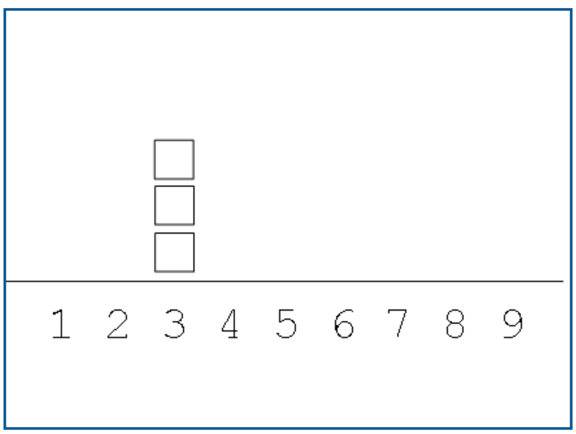
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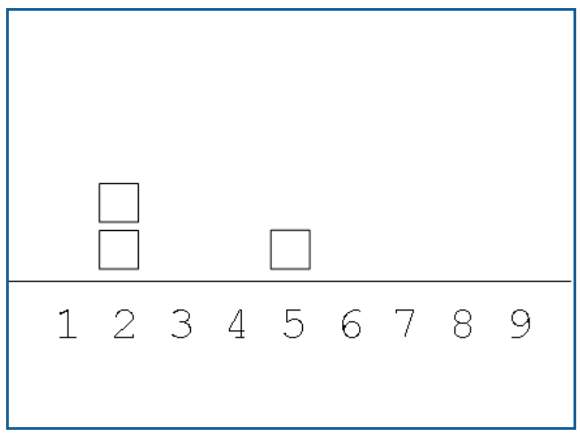
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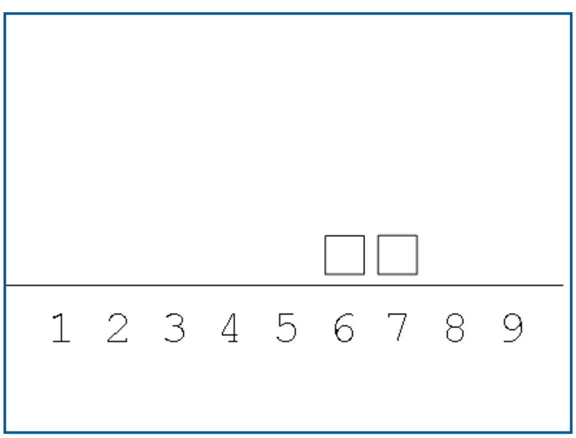
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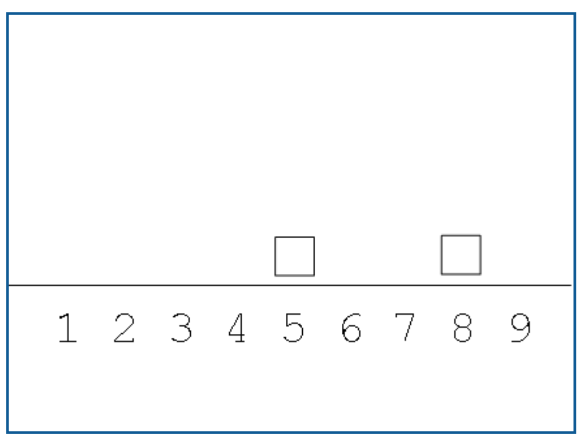
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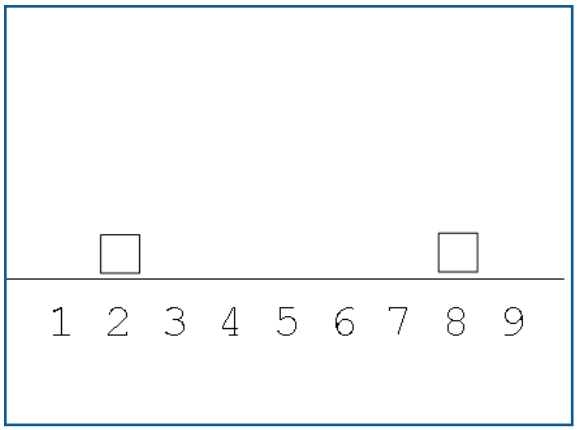
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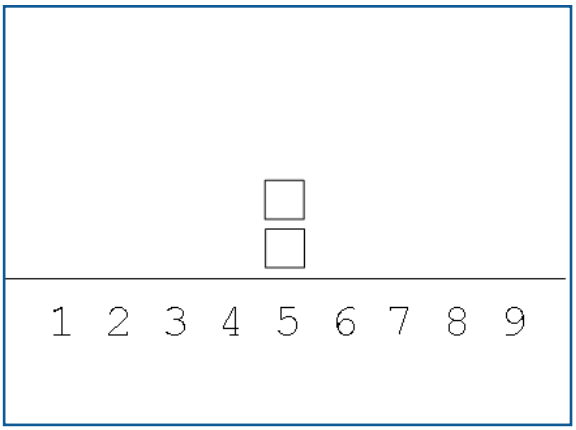
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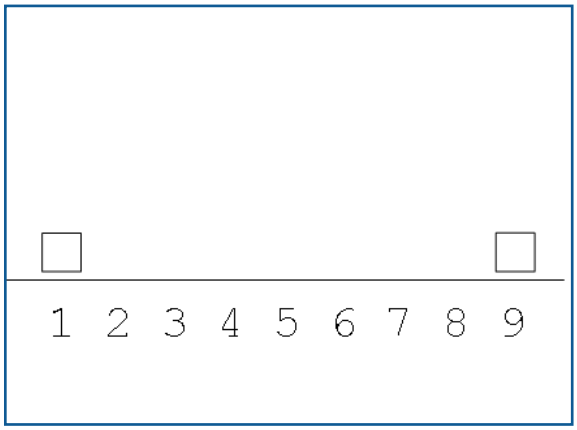
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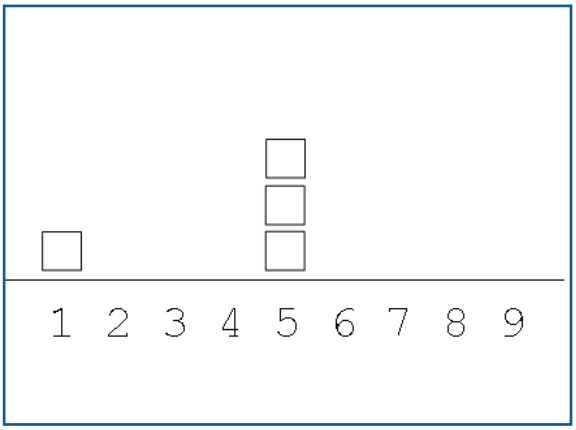
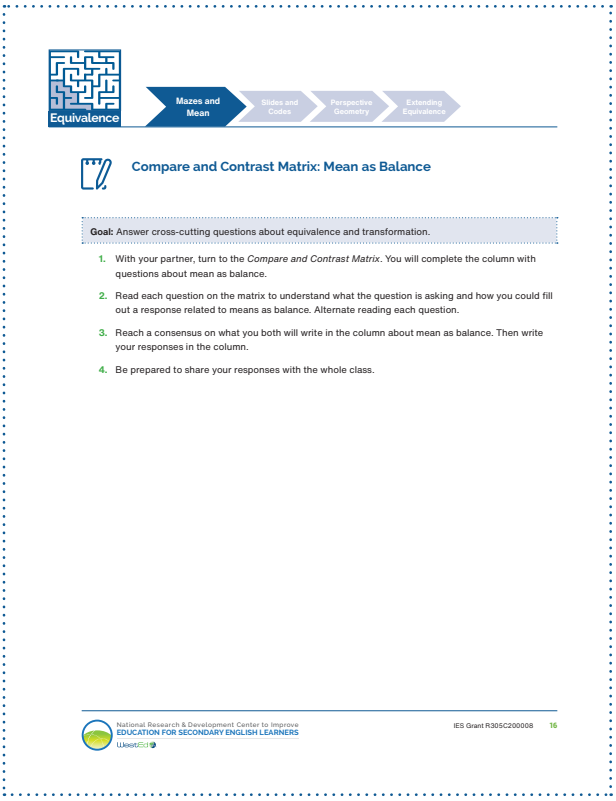

1 2 3 4 5 6 7 8 9

Compare and Contrast Matrix: Mean as a Point of Balance

Purpose and Rationale

The purpose of this task is for students to identify the concepts of equivalence and transformations found within the *Mean as a Point of Balance* task. When students compare different task and identify what transformations are allowed and not allowed based on their individual properties, they will reinforce their understanding of equivalence and therefore be able to communicate what the concept of equivalence is.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Mean as Balance

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about mean as balance.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to means as balance. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about mean as balance. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

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Prepare

1. Organize students in pairs. Distribute one copy of the *Compare and Contrast Matrix* to each student.
2. Have each student read aloud each question on the first column to understand the question and how the matrix should be completed. Partners should alternate reading the questions.

Enact and Observe

3. Students will write individually in the column about mean as point of balance. Partners will share their responses and reach a consensus.
4. Remind students that they should be prepared to share what their partners have discussed.

Close and Connect

5. Once they have completed the matrix, hold a whole class discussion where you elicit participation from the class to share what they found to be similar and different among the problems.

Reflect After Day 1

After the last activity of Day 1 has concluded, please reflect on student learning by responding to the following questions:

1. What did you notice about how students shared their reasoning about the statements in the *Think Pair Share* activity? In what ways did they use the concept of equivalence to help them compare and contrast the different images?

2. For which students did the *Human Number Line* activity work well? How do you know?

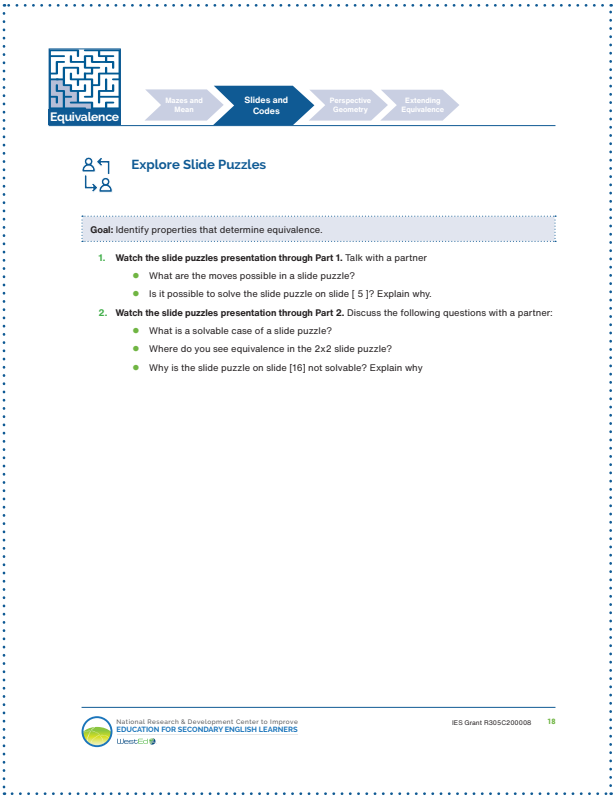
3. What key understandings about equivalence as it relates to mean did you notice students have during the *Sort and Label* activity? What did you learn about your students' understandings when you noticed how they made these connections to equivalence?

4. The *Step it Up* activity is an activity that students completed with their peers that had them engage in the process of transforming the data while preserving the mean. What, if any, challenges emerged during that activity, and how will you address them when students engage in future tasks that will also have them explore equivalence and transformations?

Explore Slide Puzzles

Purpose and Rationale

The purpose of this task is for students to identify the concepts of equivalence and transformations found within puzzles. This will help them develop a deeper understanding of equivalence and provide them the tools necessary to explain the properties found within this concept.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Explore Slide Puzzles

Goal: Identify properties that determine equivalence.

1. **Watch the slide puzzles presentation through Part 1.** Talk with a partner
   - What are the moves possible in a slide puzzle?
   - Is it possible to solve the slide puzzle on slide [5]? Explain why.
2. **Watch the slide puzzles presentation through Part 2.** Discuss the following questions with a partner:
   - What is a solvable case of a slide puzzle?
   - Where do you see equivalence in the 2x2 slide puzzle?
   - Why is the slide puzzle on slide [16] not solvable? Explain why

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Prepare

1. Organize students in pairs. Direct students to the power point slide around *Exploring Slide Puzzles.*

Enact and Observe

2. Have students discuss with a partner what are the possible moves in a slide puzzle.
3. Prompt the following on the slides:
   - What is a solvable case in a slide puzzle?
   - Where do you see equivalence in a 2 x 2 puzzle?
4. As pairs are discussing their work, identify strategies being discussed in order to share them later.

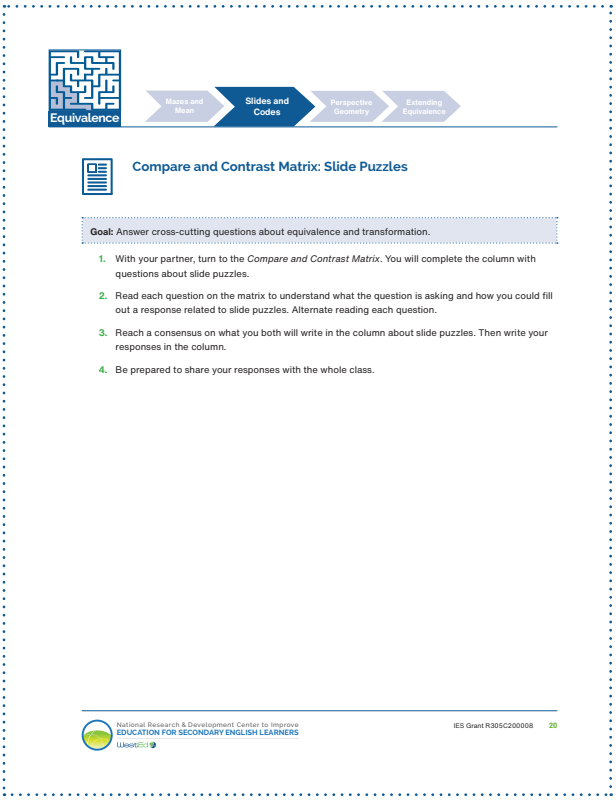
Close and Connect

5. Close the activity by holding a whole class discussion where you elicit participation from the class to share strategies, thoughts, and ideas around equivalence.

Compare and Contrast Matrix: Slide Puzzles

Purpose and Rationale

The purpose of this task is for students to identify and understand concepts of equivalence and transformations found within the slide puzzle task. When students compare different problems and identify what transformations are allowed and not allowed based on their individual properties, they will reinforce their understanding of equivalence and therefore be able to communicate what the concept of equivalence is.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Slide Puzzles

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about slide puzzles.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to slide puzzles. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about slide puzzles. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

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Prepare

1. Organize students in pairs. Distribute one copy of the ***Compare and Contrast Matrix*** to each student.
2. Have each student read aloud each question on the first column to understand the question and how the matrix should be completed. Partners should alternate reading the questions.

Enact and Observe

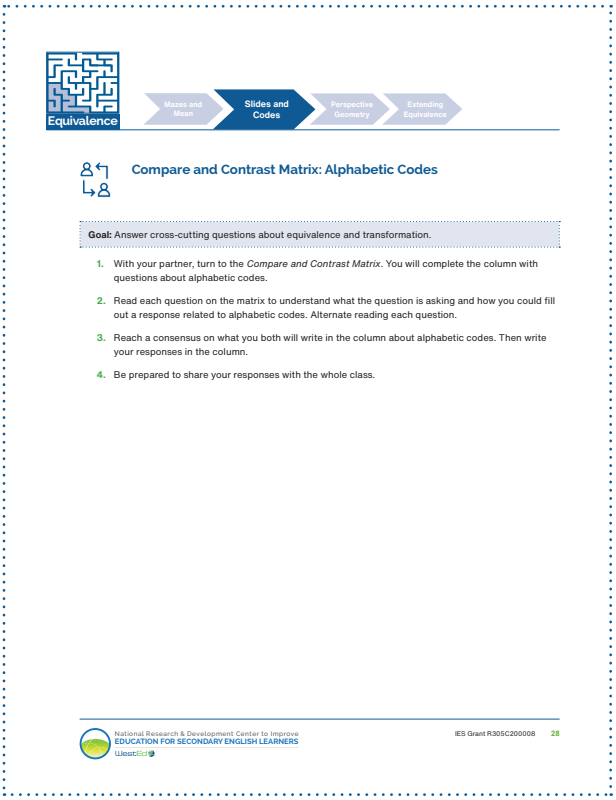
3. Students should write their answers individually, then share with a partner to Partners should reach a consensus.
4. Remind students that they should be prepared to share what their partners have discussed.

Close and Connect

5. Once they have completed the matrix, hold a whole class discussion where you elicit participation from the class to share what they found to be similar and different among the problems.

Alphabetic Codes

The *Alphabetic Codes* activity include opportunities to learn more around the concept of equivalence as it arises when discussing codes in alphabets.



Purpose and Rationale

The purpose of this activity is for students to explore how operations can be utilized to create and then transform alphabetic codes in order to make them more private.

Prepare

1. Organize students in pairs. Have students watch the screencast and consider the following question:
    - How do you keep information private?
2. Present the slides “Alphabetic Codes” in the teacher slide deck.

Enact and Observe

3. Pause the slide deck in designated places and allow students to discuss using the clarifying bookmark III.
4. Have students consider the following questions:
    - How can you undo operations used to transform the alphabet?
    - Under what conditions are operations useful for making secret codes?
    - How do you keep your information private?

Close and Connect

5. Close the activity with a whole class discussion where partners share what they have discussed around the questions presented.

Clarifying Bookmarks

Clarifying Bookmark I

What you can do	What you can say
I am going to think about what the selected text may mean.	*I'm not sure what this is about, but I think it means ...*
	This part is interesting and I think it means ...
	After rereading this part, I think it may mean ...
I am going to summarize my understanding so far.	*What I understand about this reading so far is ...*
	I can summarize this part by saying ...
	The main points of this section are ...

Clarifying Bookmark II

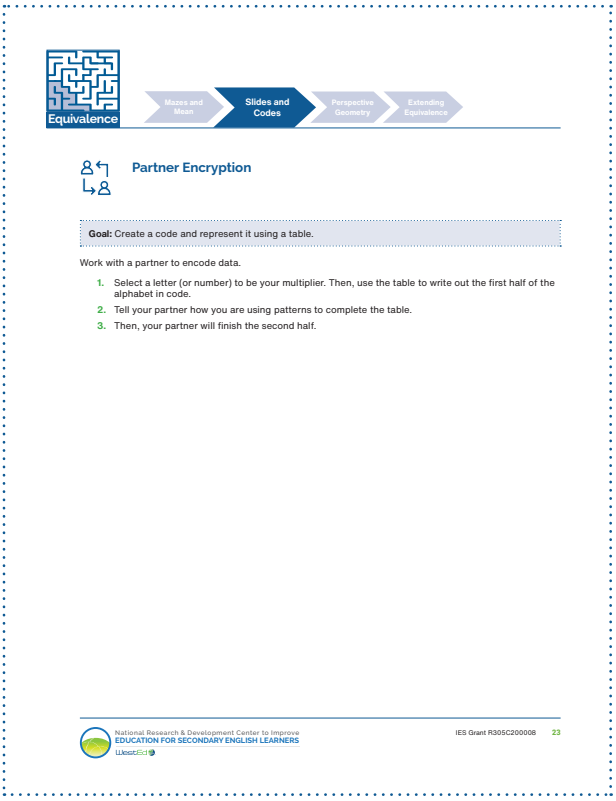
What you can do	What you can say
I am going to use my prior knowledge to help me understand.	*I know something about this from ...*
	I have read or heard about this when ...
	I don't understand the section, but I do recognize ...
I am going to apply related concepts and/or readings.	*One reading/idea I have encountered before that relates to this is ...*
	We learned about this idea/concept when we studied ...
	This concept/idea is related to ...

Clarifying Bookmark III

What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Partner Encryption

The *Partner Encryption* activity will allow students to create a code and represent it using a table.



Purpose and Rationale

The purpose of this activity is for students to deepen their understanding of equivalence by exploring how operations can be utilized to transform alphabetic codes. They will be able to systematize their alphabetic codes using a tabular representation.

Prepare

1. Have students work in partners then select a letter (or number) to be their multiplier.

Enact and Observe

2. Have students use the table to write out the first half of the alphabetic in code.
3. Students can then discuss how they are using patterns to complete the table.
4. The other partner will then complete the second half.

Close and Connect

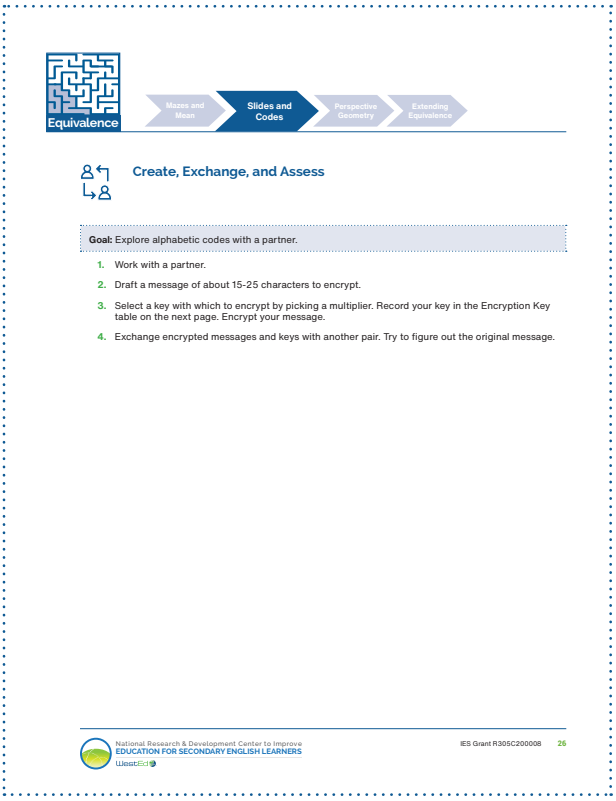
5. Hold a whole class discussion where partners share what they have discussed around the different patterns used to complete the table and around the concept of equivalence that is found within this activity.

Partner Encoding Recording Sheet

		Multiplier		Responses
A	1			
B	2			
C	3			
D	4			
E	5			
F	6			
G	7			
H	8			
I	9			
J	10			
K	11			
L	12			
M	13			
N	14			
O	15			
P	16			
Q	17			
R	18			
S	19			
T	20			
U	21			
V	22			
W	23			
X	24			
Y	25			
Z	26=0			

Create, Exchange, and Assess

The *Create, Exchange, and Assess* activity will provide partners an opportunity to further explore alphabetic codes with a partner.



Purpose and Rationale

The purpose of this task is to have students deepen their understanding of equivalence and transformations through creating and assessing alphabetic codes that were developed after discussions in their groups.

Prepare

1. Have students work in partners.
2. Have students discuss a message that will be approximately 15-25 characters in length.

Enact and Observe

3. Have partners draft the message on a sheet of paper. Partners should then discuss a key in which to encrypt by picking a multiplier. They will record their key in the *Encryption Key* table.
4. They will then encrypt their message in the recording sheet.
5. Have partners exchange with another pair and try to figure out the original message.

Close and Connect

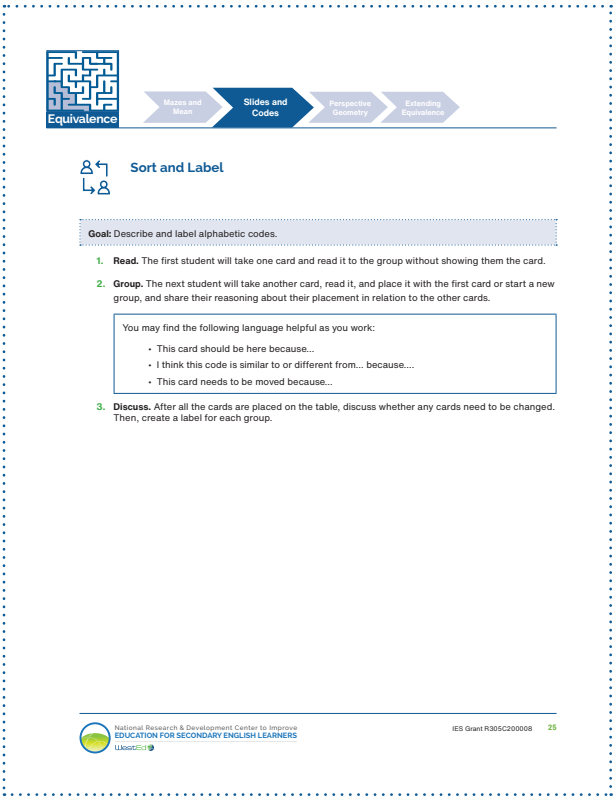
6. Have a whole class share out where partners discuss strategies they developed to try to uncover the original message. Then have the class share out how the concept of equivalence can be found within this activity.

Encryption Recording Sheet

Plain text	Plain number	Encrypted number	Encrypted letter

Sort and Label

The *Sort and Label* activity will allow students to describe and label alphabetic codes.



Purpose and Rationale

The purpose of this task is to have students describe cards that have alphabetic codes and group them according to similar characteristics that they reach a consensus around.

Prepare

1. Organize students into groups of four. Distribute the *Sort and Label Cards.*

Enact and Observe

2. Tell students that they describe one card at a time to the group without showing them the card.
3. The next student will take another card, read it, and place it with the first card or start a new group. They will explain their reasoning behind their choice. Offer the following language to support their discussions:

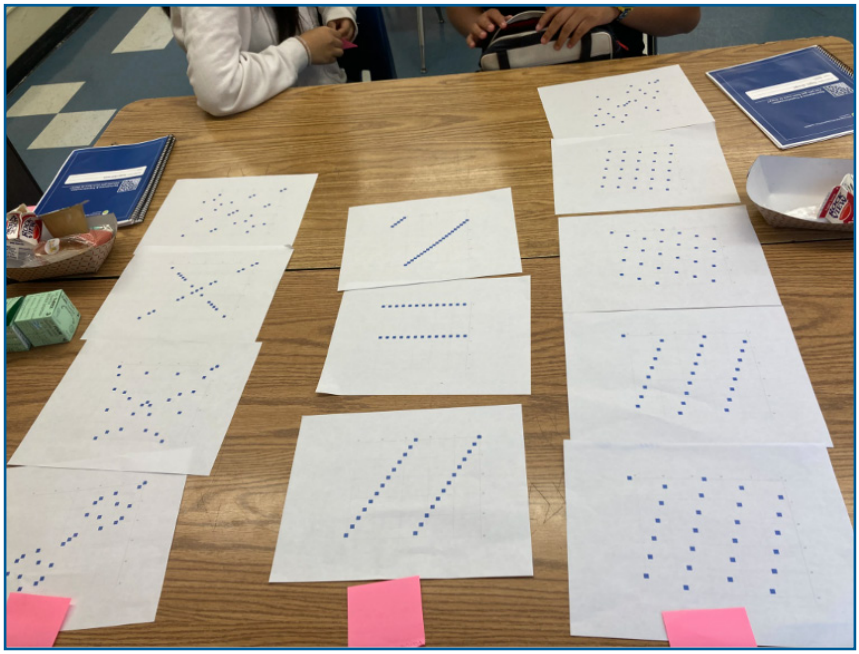
> You may find the following language helpful as you work:
>
> - This card should be here because ...
> - I think this code is similar to or different from ... because ....
> - This card needs to be moved because ...

4. After all cards have been placed on the table, guide students to discuss whether any cards need to be changed.
5. Have students create labels for their cards and be ready to share the reason behind their labels in a whole class discussion.

Close and Connect

6. Close the activity by sharing a sample of categories that were present across groups. Encourage students to with the whole class the reason behind their labels.

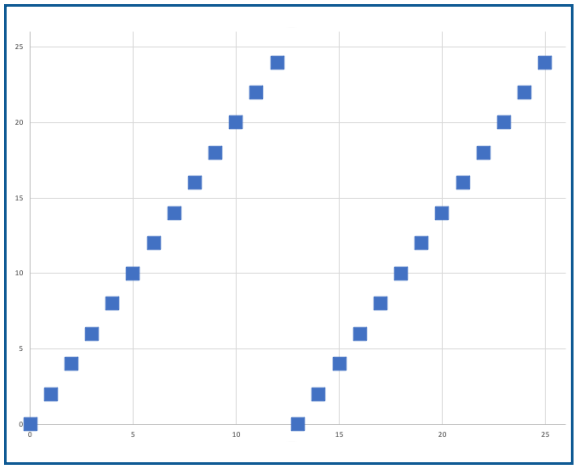
Sample Student Work and Sample Labels

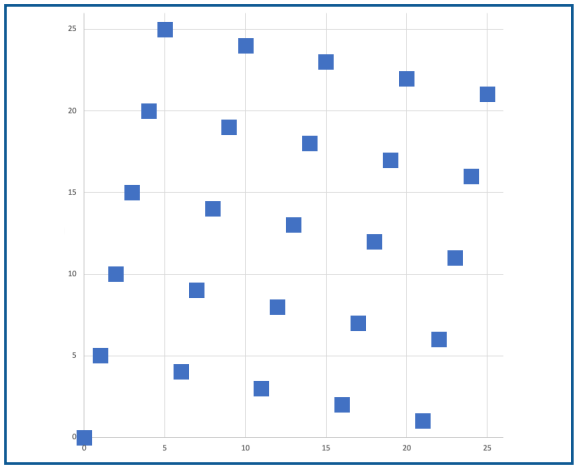


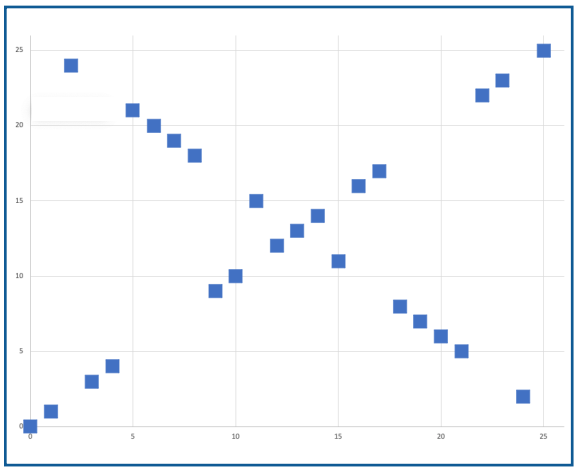
Group	Labels			
1	X's	Two parallel lines	More than two lines	No pattern
2	Doesn't start at the origin	Dot pattern	Parallel	
3	Random pictures	Forms lines	A pattern with squares	

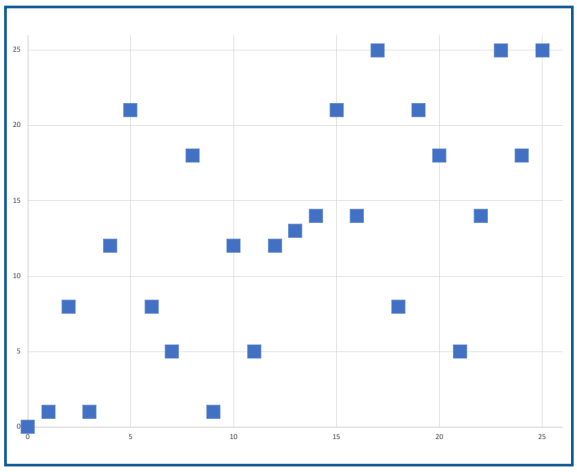
Table 1. Student-Generated Categories for Graphs of Alphabetic Codes

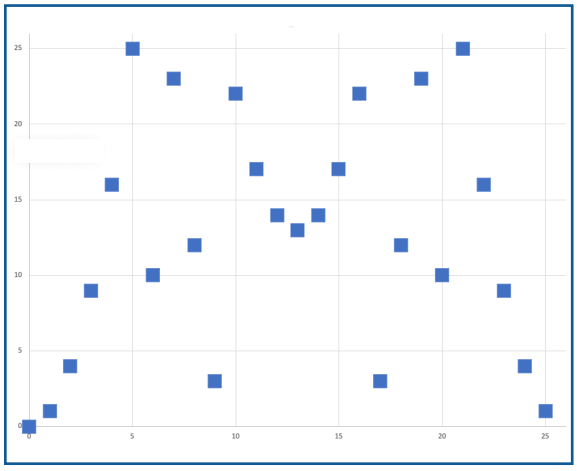
Sort and Label Cards

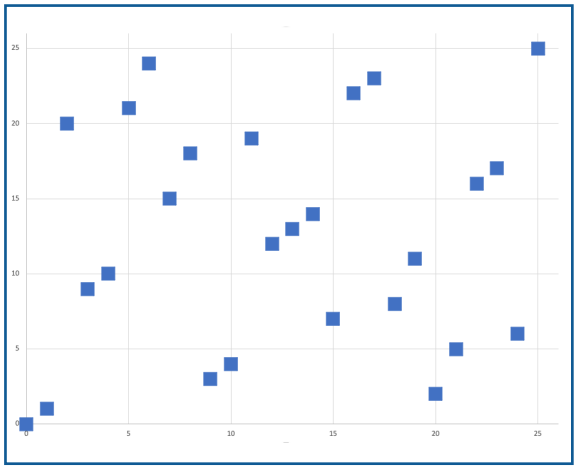


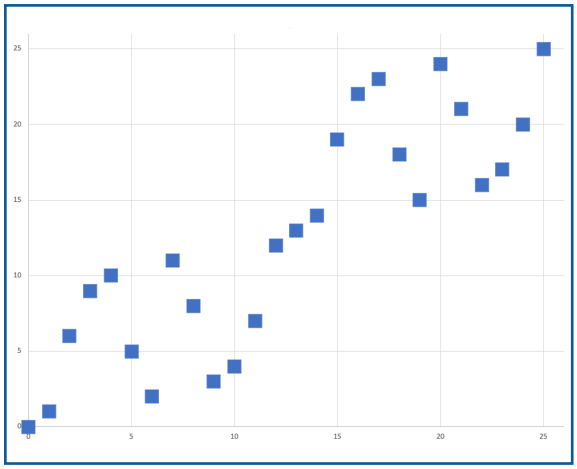


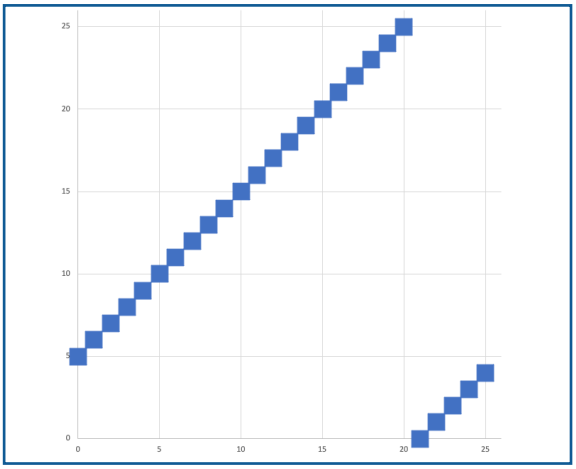


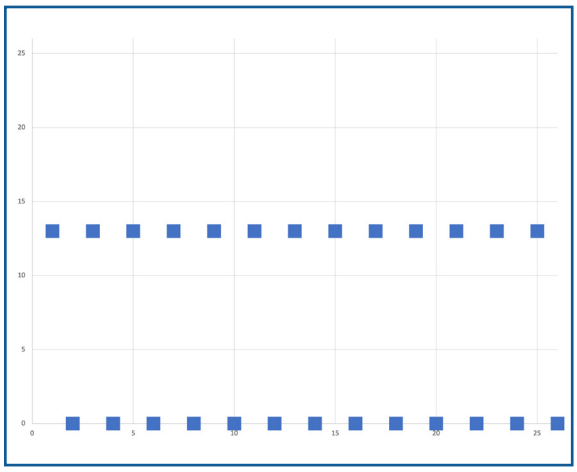


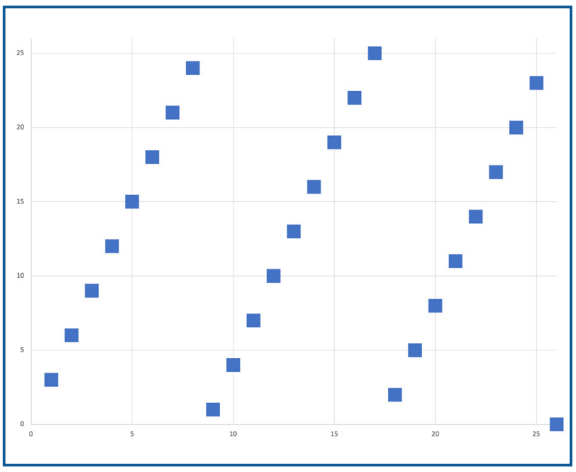


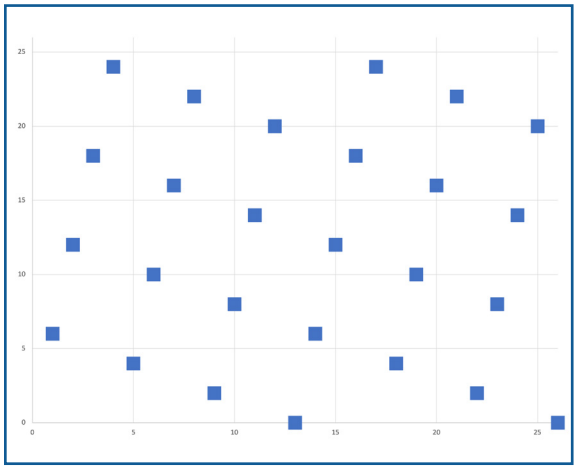


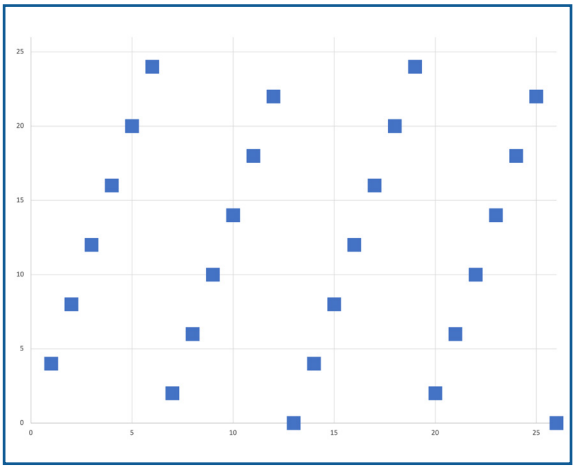








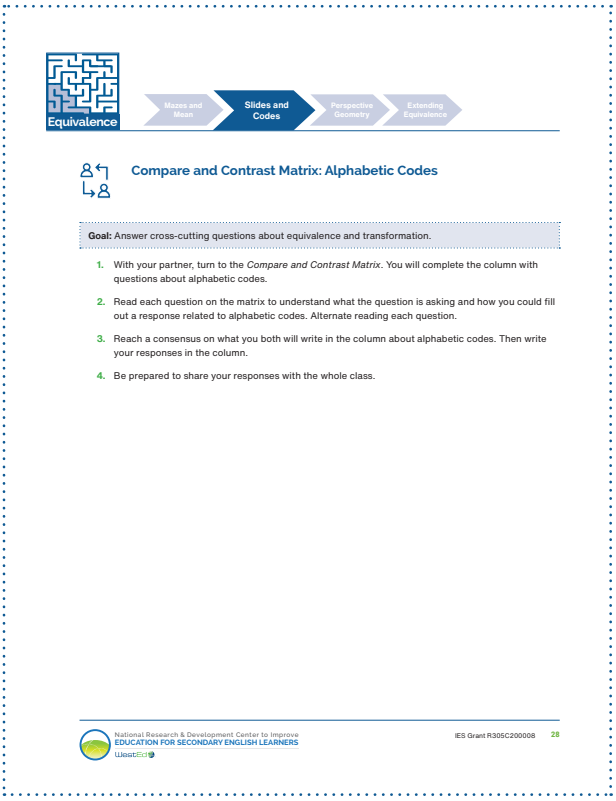




Compare and Contrast Matrix: Alphabetic Codes

Purpose and Rationale

The purpose of this task is for students to identify and understand concepts of equivalence and transformations found within the Alphabetic Codes. When students compare different problems and identify what transformations are allowed and not allowed based on their individual properties, they will reinforce their understanding of equivalence and therefore be able to communicate what the concept of equivalence is.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Alphabetic Codes

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about alphabetic codes.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to alphabetic codes. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about alphabetic codes. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

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Prepare

1. Organize students in pairs. Distribute one copy of the Compare and Contrast Matrix to each student.
2. Have each student read aloud each question on the first column to understand the question and how the matrix should be completed. Partners should alternate reading the questions.

Enact and Observe

3. Have students complete the matrix individually then discuss to reach a consensus on what should be inside each matrix box.
4. After completing each math problem, have students complete the matrix for that specific problem.

Close and Connect

5. Once they have completed the matrix, have students work in pairs, taking turns to note similarities and differences across the activities they have already engaged in.
   Provide students with formulaic expressions such as:
   - One similarity between these two problems is ...
   - These two problems are different because ....
6. Hold a whole class discussion where you illicit participation from the class to share what they found to be similar and different among the problems

Reflect After Day 2

After the last activity of Day 2 has concluded, please reflect on student learning by responding to the following questions:

1. What did you notice about how students shared their reasoning about the statements in the *Alphabetic Codes* activity? In what ways did they use the concept of equivalence to help them compare and contrast the different images?

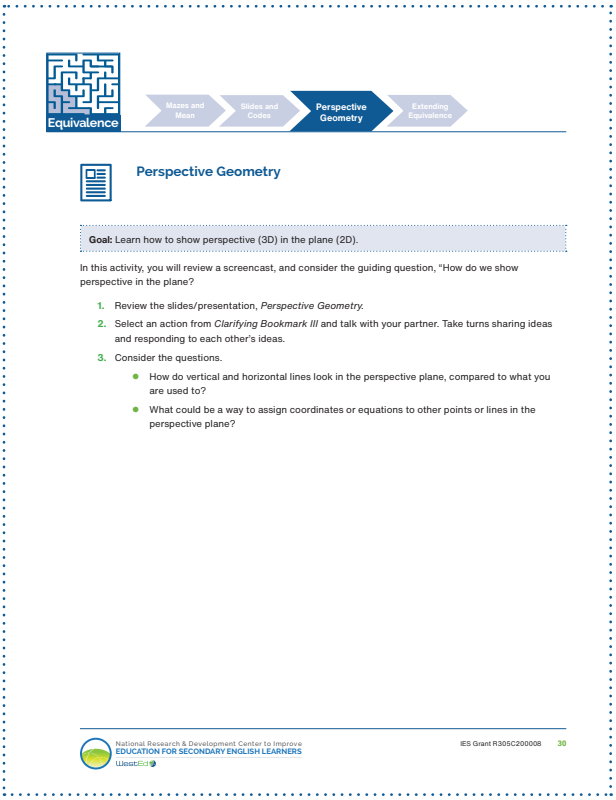
2. For which students did the *Partner Encryption* activity work well? How do you know?

3. What key understandings about equivalence did you notice students have during the *Sort and Label* activity? What did you learn about your students' understandings when you noticed how they made these connections to equivalence?

4. The *Create, Exchange, and Assess* activity is an activity that partners completed that had them engage in the process of creating their own codes. What, if any, challenges emerged during that activity, and how will you address them when students engage in future tasks that will also have them explore equivalence and transformations?

Perspective Geometry

The *Perspective Geometry* activity will allow students to learn how to show perspective (3D) in the plane (2D).



Purpose and Rationale

The purpose of this task is to have students deepen their understanding of equivalence and transformations through their application of equations in the perspective plane.

Prepare

1. Have students review the slide presentation Perspective Geometry.

Enact and Observe

2. Have partners select an action from the *Clarifying Bookmark III* and discuss with their partners. Have them take turns sharing ideas and responses.
3. Have the students consider the questions:
    - How do vertical and horizontal lines look in the perspective plane, compared to what you are used to?
    - What could be a way to assign coordinates or equations to other points or lines in the perspective plane?

Close and Connect

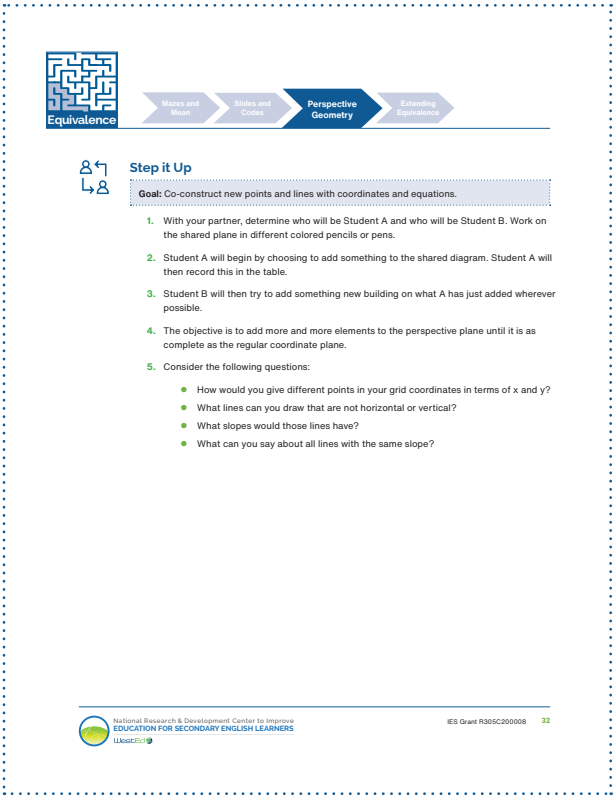
4. Have a whole class discussion where partners share what they have discussed.

Clarifying Bookmark III

What you can do	What you can say
I am going to ask questions about ideas or phrases that I do not understand.	*Two questions I have about this section are ...*
	I understand this section, but I have a question about ...
	I have a question about ...
I am going to use related text, pictures, tables, and graphs to help me understand unclear ideas.	*If we look at this graphic, it shows ...*
	The table gives me more information about ...
	When I scanned the earlier part of the reading, I found ...

Step It Up

Students will work with pairs to complete the *Step It Up* activity.



Purpose and Rationale

The purpose of this task is for students to deepen their understandings of equivalence in another context through experiencing the *Step It Up* activity. The *Step It Up* activity provides an opportunity for students to co-create transformations, highlighting ideas of equivalence across contexts.

Prepare

1. Distribute the *Step it Up* handout to pairs, then have them decide who will be partner A and partner B. Have them select a different color pencil or pen.

Enact and Observe

2. Students will complete the *Step it Up* activity in pairs A and B. They will each take turns proposing additions to the diagrams. Students will alternate and add wherever possible.
3. Have students add more elements to the perspective plane until it is as complete as the regular coordinate plane.
4. Use the following prompts as you monitor student work:
    - What elements does the coordinate plane have that you can add here?
    - Can you draw lines that are not horizontal or vertical?
    - What do you notice about lines with the same slope?

Close and Connect

5. After students complete *Step It Up* activities, prompt students to individually add to the *Compare and Contrast Matrix*. Ask students to discuss with their partner what additional ideas about equivalence they learned.
6. Have a whole group discussion where students share what their partner has stated about new learning related to equivalence. Highlight specific ways in which their discussion of equivalence related to perspective geometry is different than their discussion of equivalence in other topics.

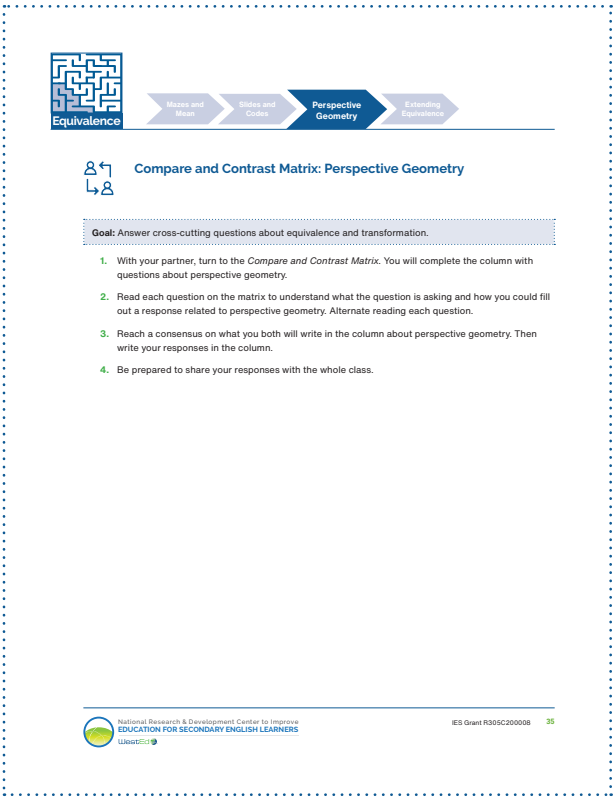
Step it Up

	What you will use	New element added
A		
B		
A		
B		
A		
B		

Compare and Contrast Matrix: Perspective Geometry

Purpose and Rationale

The purpose of this task is for students to identify and understand concepts of equivalence and transformations found within the *Perspective Geometry*. When students compare different problems and identify what transformations are allowed and not allowed based on their individual properties, they will reinforce their understanding of equivalence and therefore be able to communicate what the concept of equivalence is.


Equivalence
Mazes and Mean | Slides and Codes | Perspective Geometry | Extending Equivalence

Compare and Contrast Matrix: Perspective Geometry

Goal: Answer cross-cutting questions about equivalence and transformation.

1. With your partner, turn to the *Compare and Contrast Matrix*. You will complete the column with questions about perspective geometry.
2. Read each question on the matrix to understand what the question is asking and how you could fill out a response related to perspective geometry. Alternate reading each question.
3. Reach a consensus on what you both will write in the column about perspective geometry. Then write your responses in the column.
4. Be prepared to share your responses with the whole class.

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Prepare

1. Organize students in pairs. Distribute one copy of the *Compare and Contrast Matrix* to each student.
2. Have each student read aloud each question on the first column to understand the question and how the matrix should be completed. Partners should alternate reading the questions.

Enact and Observe

3. Have students complete the matrix individually and then discuss to reach a consensus on what should be inside each matrix box.
4. After completing each math problem, have students complete the matrix for that specific problem.

Close and Connect

5. Once they have completed the matrix, have students work in pairs, taking turns to note similarities and differences across the activities they have already engaged in. Provide students with formulaic expressions such as:
    - One similarity between these two problems is ...
    - These two problems are different because ...
6. Hold a whole class discussion where you illicit participation from the class to share what they found to be similar and different among the problems.

Reflect After Day 3

After the last activity of Day 3 has concluded, please reflect on student learning by responding to the following questions:

1. What did you notice about how students shared their reasoning about the statements in the *Perspective Geometries* activity? In what ways did they use the concept of equivalence to help them compare and contrast the different lines?

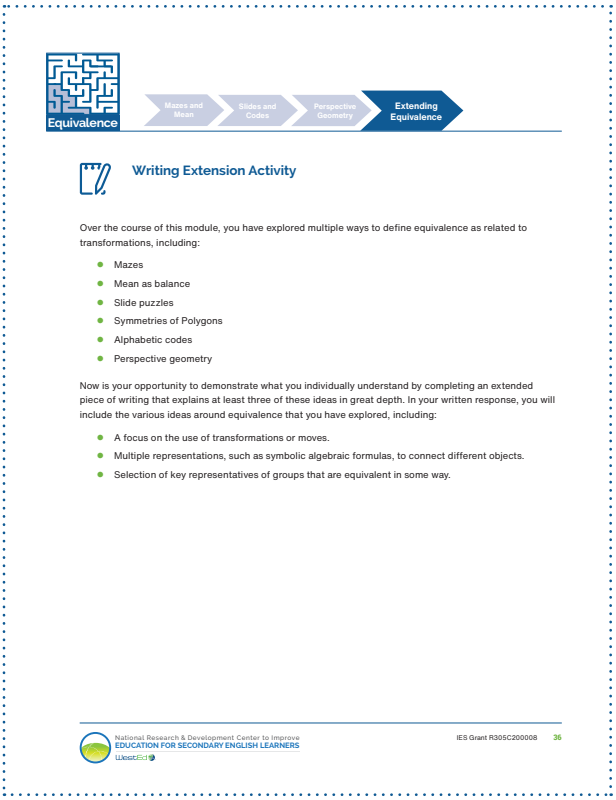
2. For which students did the *Step It Up* activity work well? How do you know?

3. What key understandings about equivalence did you notice students have during the *Step It Up* activity? What did you learn about your students' understandings when you noticed how they made these connections to equivalence?

4. What, if any, challenges emerged during the Step It Up activity?

Writing Extension Activity

Students will reflect and demonstrate their understanding around equivalence through a writing activity.



Purpose and Rationale

The purpose of this task is for students to synthesize their understandings of equivalence across a broad variety of cases in algebra, geometry, and statistics.

Prepare

1. Have students turn to the *Writing Extension Activity Rubric* in their binders.
2. Go over the sections coving the *Content and Design* expectations.

Enact and Observe

3. Have students develop their writing responses around the following:
    - A focus on the use of transformations or moves.
    - Multiple representations, such as symbolic algebraic formulas, to connect different objects.
    - Selection of key representatives of groups that are equivalent in some way.

Close and Connect

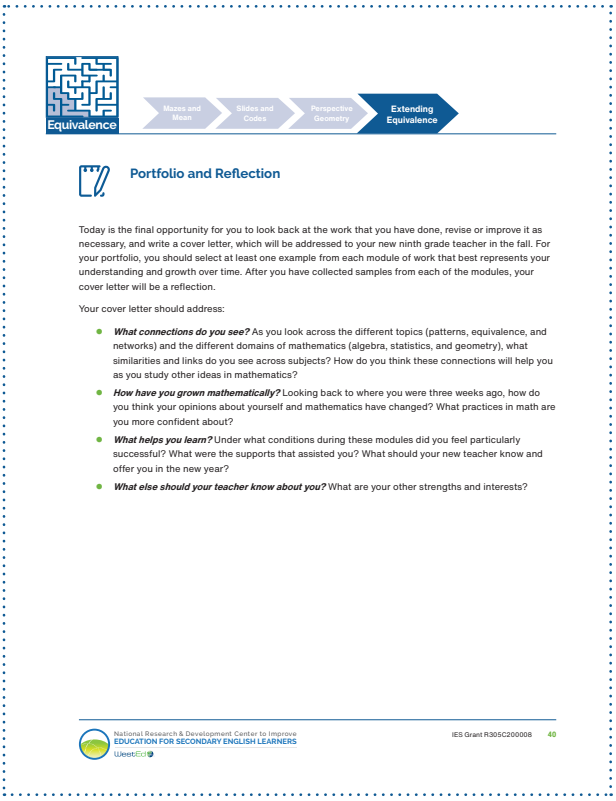
4. Have students share ideas they are developing to each other to help refine their draft.

Writing Extension Activity Rubric

Category	Indicators of High Quality Work	Strengths	Areas to Improve or Revise
Content	• Clearly unpacked examples illustrate key ideas and offer a general approach that could be extended to other examples. • General statements of fact and demonstrations of statements that are true or not true in different cases, with justifications for why these statements should be true. • Explanations of systems provide a rule or set of criteria for distinguishing different objects or proving that they are equivalent. • Explanations explicitly compare and contrast cases in terms of statements or other models, such as algebraic ones.		
Design	• Use of visual images and color is effective for adding to the meaning communicated. • Elements combine to show or highlight connections.		

Portfolio and Reflection

Students will reflect on their summer experience and look forward to their high school career.



Purpose and Rationale

The purpose of this task is for students to reflect on who they are and what they have learned this summer in a letter to their new high school teacher.

Prepare

1. Have students turn to the *Portfolio and Reflection* in their binders.
2. Go over the sections covering the content expectations.

Enact and Observe

3. Have students develop their writing responses around the following:
    - What connections they see in the different topics and domains of mathematics
    - How they have grown mathematically
    - What helps them learn
    - What are their strengths and interests

Close and Connect

4. Have students share ideas they are developing to each other to help refine their draft.

Reflect After Day 4

Please reflect on student learning by responding to the following questions:

1. What strategies did students use for the *Writing Extension* task? Were any of their strategies unexpected?

2. For which students did the *Compare and Contrast Matrix* facilitate their planning for their writing? How do you know?

3. What, if any, changes in student mathematical talk did you notice across the module?

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